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A Method for Reliability Allocation with Confidence Level

Huairui Guo, PhD, ReliaSoft Corporation

Mingxiao Jiang, PhD, Medtronic, Inc.

Wendai Wang, PhD, Thoratec Corporation.

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SUMMARY & CONCLUSIONS

One of the important reliability activities in Design for Reliability (DFR) is system reliability allocation at the early product design stage. Usually system reliability is given as a product performance requirement under the normal use condition (e.g., the probability of failure for a 2-year operation should be less than 0.05 with a confidence level of 90%). Complex systems consist of many subsystems, which are developed concurrently and sometimes independently. It would be too late to validate the system reliability until the final system prototype is ready after months or years of development. From a project management point of view, the reliability for each subsystem or sub-function should be examined as early as possible. Therefore, allocating a reasonable reliability requirement to each subsystem and sub-function based on the system reliability requirement is very important. Many reliability allocation approaches can be used, such as the AGREE, weighted, equal reliability, and cost-based methods. Traditional system reliability allocation is conducted on the reliability requirement alone. None of the above existing methods considers allocating the reliability requirement together with a confidence level.

In this paper, we will propose a new approach for allocating system reliability together with confidence level to the subsystems. The proposed method can be used for complex systems with serial, parallel, and k -out-of- n configurations.

1 INTRODUCTION

The development cycle for many systems such as medical devices and military equipment usually is very long. The system reliability requirement should be allocated to each subsystem as early as possible in the product development stage. If each subsystem cannot meet its own reliability requirement, then the final system will not be able to meet the requirement. Due to this reason, many reliability allocation methods have been proposed in the past half century [1]. The simplest reliability allocation method is the so called *equal reliability allocation*. This method is used for a system with a serial reliability configuration. For example, assume a system has three independent subsystems A, B, and C. The system reliability is a function of the subsystem reliability, which is:

$$R_S = R_A \times R_B \times R_C \quad (1)$$

where R_S is the system reliability. If the required R_S is 0.90, the reliability of each subsystem will be:

$$R_A = R_B = R_C = (R_S)^{1/3} = 0.9655 \quad (2)$$

Once one of the subsystems is ready for testing, we can design a demonstration test to see if it meets the required reliability. For example, 30 samples for subsystem A are tested and 1 failure was obtained; does it meet the required reliability of 0.9655? The estimated reliability can be calculated as:

$$\hat{R}_A = 29/30 = 0.9667 \quad (3)$$

The above result shows that subsystem A meets the requirement. However, since R_A is estimated from the data, there is uncertainty associated with the \hat{R}_A . A large sample size will reduce the uncertainty of \hat{R}_A . The variance of \hat{R}_A is:

$$Var(\hat{R}_A) = \hat{R}_A(1 - \hat{R}_A)/n \quad (4)$$

Eqn. (3) is the so called *Maximum Likelihood Estimate* (MLE). Using Eqn. (3) and Eqn. (4), the confidence bounds of R_A at a given confidence level can be calculated [2].

Now if 20 samples are tested without any failure, does subsystem A meet the requirement? Clearly, the calculation in Eqn. (3) and (4) cannot be used when there is no failure. Otherwise, the estimated reliability will be 1 without any uncertainty since its variance is 0. Instead of using the MLE estimator, the following binomial equation is used for reliability estimation:

$$1 - CL = \sum_{i=0}^r \binom{n}{i} (1 - R)^i R^{n-i} \quad (5)$$

where n is the sample size, r is the number of failures and CL is the confidence level. We know the sample size n is 20 and the number of failures is 0. So using these values, Eqn. (5) becomes:

$$1 - CL = R^n \quad (6)$$

Without knowing a value for CL , we cannot solve R from Eqn. (6). If we set $CL = 0.5$, then we have:

$$1 - CL = R^n \Rightarrow 0.5 = R^{20} \Rightarrow R = 0.9659 \quad (7)$$

So at a confidence level of 0.5, the reliability meets the requirement of 0.9655. However, if we set $CL = 0.95$, then:

$$1 - CL = R^n \Rightarrow 0.05 = R^{20} \Rightarrow R = 0.8609 \quad (8)$$

The result shows that the reliability does not meet the requirement. Therefore, without giving a confidence level in the reliability requirement, we cannot answer the question: "Does A meet the reliability requirement?". The confidence level must be in the reliability requirement.

However, none of the existing reliability allocation methods can be applied to allocate the reliability with a confidence level. Very little research has been done on allocating system reliability requirement with a confidence level to subsystems. In this paper, we will develop a new method to solve this important issue. The paper is arranged as follows: Section 2 will briefly review several commonly used reliability allocation methods first and then we will explain our new method. Section 3 will give two examples to illustrate how to apply the proposed method. Section 4 is the conclusion.

2 SYSTEM RELIABILITY ALLOCATION WITH CONFIDENCE LEVEL

From the above section, we can see that the system reliability is a function of its subsystem reliability. For a system with k subsystems/components, its reliability is:

$$R_s = g(R_1, R_2, \dots, R_k) \quad (9)$$

where:

- R_s is the system reliability.
- $R_i (i = 1, \dots, k)$ is the reliability of subsystem i .
- g is the function between R_i and R_s .

If the system reliability requirement R_s^* is given, we need to find R_i to make sure:

$$R_s^* \leq g(R_1, R_2, \dots, R_k)$$

Several methods for reliability allocation have been developed [1]. The simplest method is the equal allocation method we illustrated in Section 1. This method can only be applied when the system reliability configuration is in series. The system reliability is calculated by:

$$R_s = \prod_{i=1}^k R_i \quad (10)$$

The allocated reliability for each subsystem is:

$$R_i = (R_s^*)^{1/k} \quad (11)$$

Other methods such as the AGREE method, the ARINC method developed by the Advisory Group on Reliability of Electronic Equipment, and Penalty Function Based Allocation are also used. For detail, please refer to [1, 3]. None of the existing allocation methods consider confidence level. We will explain our allocation method for system reliability and

confidence level next.

As discussed in Section 1, when reliability is estimated from data, the estimated reliability is a random variable which can be described by a distribution. For example, when the MLE estimate is used, \hat{R}_i is assumed to be normally distributed asymptotically, with mean of $(1-r_i)/n_i$ and variance as given in Eqn. (4), where r_i is the number of failures when n_i samples are tested [2]. When the binomial equation, Eqn. (5), is used to estimate the reliability, the estimated reliability \hat{R}_i follows a beta distribution $Beta(R_i; n_i - r_i, r_i + 1)$ [4]. If the CL percentile of \hat{R}_i is greater than R_i^* , a reliability requirement at confidence level CL , then it means the reliability has met the requirement.

2.1 Reliability Requirement with a Confidence Level

The binomial formula of Eqn. (5) is commonly used in reliability demonstration test (RDT) design. It has four variables: R , CL , sample size n , and number of failures r . Since R and CL are given as part of the requirement, we need to find n and r in designing a demonstration test. For example, if the requirement is $R = 90\%$ and $CL = 90\%$, then there are an infinite number of combinations of n and r that can demonstrate the required reliability. Some of the combinations are given in Table 1.

Number of Failures	Sample Size
0	22
1	38
2	52
3	65
4	78
5	91

Table 1: Test Plan for $R=90\%$ and $CL=90\%$

From the above table, we can see that testing 22 samples without failure is equivalent to testing 38 samples with 1 failure in terms of demonstrating $R = 90\%$ and $CL = 90\%$. The zero failure test requires the smallest sample size, so it is the most popularly used test in RDT.

The system reliability is a function of the subsystem reliabilities, which are estimated from test data. Since the distribution of reliability is determined by the sample size and the number of failures, the reliability allocation problem is the same as determining the test plan for each subsystem. In other words, we need to calculate the system reliability at a given confidence level from the estimated subsystem reliability.

We will discuss how to calculate the system reliability distribution from subsystem reliability in the next section.

2.2 Mean and Variance for Component and System Reliability

The estimated reliability from Eqn. (5) for the i th component or subsystem is a beta distribution $Beta(\hat{R}_i; n_i - r_i, r_i + 1)$. The mean and variance of \hat{R}_i are:

$$E(\hat{R}_i) = (n_i - r_i) / (n_i + 1) \quad (12)$$

$$Var(\hat{R}_i) = [(n_i - r_i)(r_i + 1)] / [(n_i + 1)^2 (n_i + 2)] \quad (13)$$

1) Serial System Reliability

The reliability of a serial system is given in Eqn. (10). Since R_i follows the beta distribution, R_s is the product of multiple independent beta random variables. The exact distribution for R_s can be found using the Mellin transform [5]. However, this calculation is very intensive. To simplify the calculation, Thompson and Haynes [6] suggested approximating the exact distribution with a beta distribution having the same first two moments. In fact, without getting the exact or approximated distribution for R_s , its mean and variance can be calculated by [7]:

$$E(\hat{R}_s) = \prod E(\hat{R}_i) = \prod \frac{a_i}{a_i + b_i} \quad (14)$$

$$Var(\hat{R}_s) = E(\hat{R}_s) \prod \frac{a_i + 1}{a_i + b_i + 1} - E^2(\hat{R}_s) \quad (15)$$

where $a_i = n_i - r_i$, $b_i = r_i + 1$.

Eqn. (15) is based on the following conclusion. If $X = \prod_{i=1}^k X_i$ and the mean and the variance for X_i are known, the variance for X can be estimated by [7]:

$$Var(X) = \prod_{i=1}^k [E^2(X_i) + Var(X_i)] - \prod_{i=1}^k E^2(X_i) \quad (16)$$

2) Parallel System Reliability

The reliability for a parallel system is calculated by:

$$R_s = 1 - \prod_{i=1}^k (1 - R_i) = 1 - \prod_{i=1}^k F_i \quad (17)$$

Since $R_i \sim Beta(n_i - r_i, r_i + 1)$ and $F_i = 1 - R_i$. F_i is also a beta distribution, $F_i \sim Beta(F_i; r_i + 1, n_i - r_i)$. Formulas similar to Eqn. (14) and (15) can be used to get the mean and variance for R_s for a parallel system. They are:

$$E(\hat{R}_s) = 1 - \prod_{i=1}^k E(\hat{F}_i) = 1 - \prod_{i=1}^k \frac{r_i + 1}{n_i + 1} \quad (18)$$

$$Var(\hat{R}_s) = \left[1 - E(\hat{R}_s) \right] \left[\prod_{i=1}^k \frac{a_i + 1}{a_i + b_i + 1} - 1 + E(\hat{R}_s) \right] \quad (19)$$

where $a_i = r_i + 1$, $b_i = n_i - r_i$.

Eqn. (19) is based on the following conclusion. If $X = 1 - \prod_{i=1}^k (1 - X_i)$, the variance for X can be estimated by:

$$Var(X) = \prod_{i=1}^k [E^2(1 - X_i) + Var(X_i)] - \prod_{i=1}^k E^2(1 - X_i) \quad (20)$$

3) Complex System

A complex system is a system that can be composed of many series and parallel subsystems. By using the above

equations iteratively for each subsystem, the system $E(\hat{R}_s)$ and $Var(\hat{R}_s)$ can be obtained. For detail, please see [4].

2.3 System Reliability Distribution Approximation

Once the $E(\hat{R}_s)$ and $Var(\hat{R}_s)$ are obtained, we will approximate the system reliability distribution by a beta distribution, $Beta(R_s; a, b)$. For a random variable X following a beta distribution with parameters a and b , its mean and variance are:

$$E(X) = a / (a + b) \quad (21)$$

$$Var(X) = ab / [(a + b)^2 (a + b + 1)] \quad (22)$$

Setting Eqn. (21) and (22) to the estimated mean and variance obtained from the previous procedure for serial, parallel and complex systems, the parameters a and b can be estimated using:

$$a = E(\hat{R}_s) \left(\left[E(\hat{R}_s) - E^2(\hat{R}_s) \right] / Var(\hat{R}_s) - 1 \right) \quad (23)$$

$$b = (1 - E(\hat{R}_s)) \left(\left[E(\hat{R}_s) - E^2(\hat{R}_s) \right] / Var(\hat{R}_s) - 1 \right) \quad (24)$$

Once a and b are found, the one-sided 100%CL lower bound for the system reliability R_{sL} can be calculated by:

$$\int_0^{R_{sL}} Beta(R_s; a, b) dR_s = 1 - CL \quad (25)$$

2.4 A General Method for System Reliability Allocation with Confidence Level

Given that the system reliability requirement is R_s^* at a confidence level of CL , the calculated R_{sL} from Eqn. (25) should be greater than R_s^* . Since the system reliability is a function of subsystem reliabilities, there are many different ways that reliability can be allocated without adding any constraints. In this paper, the following constraints are considered in the allocation:

- The confidence level for each subsystem is the same as the one for the system.
- All of the system and subsystem tests are zero-failure tests.

The above constraints fix $CL_i = CL$ and $r_i = 0$. In the binomial equation:

$$1 - CL_i = \sum_{i=0}^{r_i} \binom{n_i}{i} (1 - R_i)^i R_i^{n_i - i} \quad (26)$$

only n_i and R_i are free to change. Allocating R_i is the same as determining n_i , since they have the one-to-one relation when CL_i and r_i are fixed.

For different subsystems, the risk of a failure and the cost of testing a sample are different. Risk can be defined as the product of failure probability and severity of the failure consequence. For example, if a subsystem failure of a medical device will cause permanent injury to a patient, then its

severity is higher than a failure that only causes minor consequences (e.g. recoverable injury or inconvenience to patient) Subsystems with high risk should be allocated with higher reliability. The cost of conducting a test for one subsystem may be more expensive or take much longer time than other subsystems. If a very high reliability value is assigned to this subsystem, it may be impossible to demonstrate it, so the allocated reliability for each subsystem should be able to demonstrate. All such important constraints should be considered in reliability allocation. The cost or risk can be the penalty function in the reliability allocation. Therefore, the reliability allocation problem becomes:

Minimize: A penalty function

Determine: R_i at confidence level of CL

st:

- $R_{sL} \geq R_s^*$
- Constraints on sample size of each subsystem, risk weighting factor, budget, etc.

Where: $R_{sL} = Inv_Beta(1-CL, a, b)$,

$$a = f_a(R_1, R_2, \dots, R_k), \quad b = f_b(R_1, R_2, \dots, R_k).$$

$a = f_a(R_1, R_2, \dots, R_k)$ and $b = f_b(R_1, R_2, \dots, R_k)$ show that a and b for the beta distribution of the system reliability are functions of the subsystem reliabilities.

2.5 A Simple Allocation Method for Serial Systems

When the system reliability configuration is in series, the above allocation procedure can be simplified by adding some constraints. For example, we can require equal reliability at the same confidence level for each subsystem. The allocation problem becomes:

Minimize: R_{sL}

Determine: R_i at confidence level of CL

st: $R_1 = R_2 = \dots = R_k, \quad R_{sL} \geq R^*$

Where: $R_{sL} = Inv_Beta(1-CL, a, b)$,

$$a = f_a(R_1, R_2, \dots, R_k), \quad b = f_b(R_1, R_2, \dots, R_k).$$

To demonstrate the same reliability at the same confidence level for a zero-failure test, the number of test samples should be the same. Therefore, in Eqn. (14) and (15), we can set $n_i = n$. Then we have:

$$E(\hat{R}_s) = (n/(n+1))^k \quad (27)$$

$$Var(\hat{R}_s) = (n/(n+2))^k - (n/(n+1))^{2k} \quad (28)$$

where n is the required samples for demonstrating the allocated reliability of each subsystem and k is the number of subsystems/components in a serial system. Substituting Eqn. (27) and (28) in Eqn. (23) and (24), we get:

$$a = \left(\frac{n}{n+1}\right)^k \left[\frac{(n/(n+1))^k - (n/(n+2))^k}{(n/(n+2))^k - (n/(n+1))^{2k}} \right] \quad (29)$$

$$b = \left(1 - \left(\frac{n}{n+1}\right)^k\right) \left[\frac{(n/(n+1))^k - (n/(n+2))^k}{(n/(n+2))^k - (n/(n+1))^{2k}} \right] \quad (30)$$

It can be seen that the optimization is much simpler since it only needs to solve for one variable, n . We need to find n in order to make $Inv_Beta(1-CL, a, b)$ equal to R_s^* as close as possible.

Assuming that the required system reliability R_s^* is 0.95 at a confidence level of 90%, the following table gives the optimization results of the allocated reliability for each subsystem for systems with different numbers of subsystems.

Number of Subsystems	Allocated R at CL=90%	Allocated R w/o CL
2	0.9701	0.9747
3	0.9781	0.9830
4	0.9825	0.9873
5	0.9853	0.9898
6	0.9873	0.9915
7	0.9888	0.9927
8	0.9900	0.9936

Table 2- Allocated Reliability with Confidence Level for Serial Systems with Different Numbers of Subsystems

The last column in Table 2 is the result from the traditional equal reliability allocation without considering matching the confidence level requirement. For this example, the traditional equal allocation method gives more conservative results than the one from the method in this paper. For example, for a system with 8 subsystems, the table shows that traditional allocation requires each subsystem's reliability to be 0.9936. The result using the proposed method is a reliability of 0.9900 at a confidence level of 90%.

3 ILLUSTRATIVE EXAMPLES

The following system will be used for illustration.

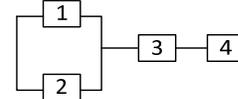


Figure 1- System with Serial and Parallel Subsystems

The required system reliability is 95% at a confidence level of 90%. Determine the required reliability for each subsystem at a confidence level of 90%. Assume a zero-failure test is required for demonstrating the reliability for each subsystem.

First, we need to decompose the system into a simple serial system. It is

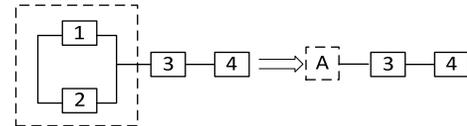


Figure 2- System Decomposition

The mean and the variance of the reliability of block A will be from Eqn. (18) and Eqn. (19). They are:

$$E(\hat{R}_A) = 1 - \prod_{i=1}^2 E(\hat{F}_i) = 1 - \prod_{i=1}^2 \frac{n_i + 1}{n_i + 1} = 1 - \frac{1}{(n_1 + 1)(n_2 + 1)} \quad (31)$$

$$Var(\hat{R}_A) = \left[1 - E(\hat{R}_A) \right] \left[\frac{4}{(n_1 + 2)(n_2 + 2)} - 1 + E(\hat{R}_A) \right] \quad (32)$$

The system reliability is:

$$E(\hat{R}_s) = E(\hat{R}_A)E(\hat{R}_3)E(\hat{R}_4) = \frac{n_1 + n_2 + n_1 n_2}{(n_1 + 1)(n_2 + 1)} \frac{n_3}{n_3 + 1} \frac{n_4}{n_4 + 1} \quad (33)$$

From Eqn. (15), the variance of the estimated system reliability is:

$$Var(\hat{R}_s) = \prod_{i \in \{A,3,4\}} [E^2(\hat{R}_i) + Var(\hat{R}_i)] - \prod_{i \in \{A,3,4\}} E^2(\hat{R}_i) \quad (34)$$

where $E(\hat{R}_A)$ and $Var(\hat{R}_A)$ are given in Eqn. (31) and (32), $E(\hat{R}_i) = n_i / (n_i + 1)$, and $Var(\hat{R}_i) = n_i / [(n_i + 1)^2 (n_i + 2)]$ for $i = 3, 4$ based on Eqn. (12) and (13).

From $E(\hat{R}_s)$ and $Var(\hat{R}_s)$, the distribution parameters a and b for \hat{R}_s are calculated from Eqn. (23) and (24). From Eqn. (25), we know:

$$\int_0^{R_{sL}} Beta(R_s; a, b) dR_s = 1 - CL = 0.1 \quad (35)$$

a and b are functions of n_i . We need to solve for n_i using Eqn. (35) to make R_{sL} as close to the required reliability R_s^* as possible. Once n_i is found, for a zero-failure test, \hat{R}_i at a given confidence level CL_i can be calculated from:

$$\hat{R}_i = \exp(\ln(1 - CL_i) / n_i) \quad (36)$$

In allocating reliability to each subsystem, several practical factors should be considered. 1) Enough samples, cost, and test time should be allowed for each subsystem to demonstrate the allocated reliability requirement. 2) High risk subsystems should have higher reliability. In the following, we will use two examples to illustrate how to allocate reliability with these constraints.

3.1 Cost Based Allocation

For the example given in Figure 1, the system reliability is required to be 95% at a confidence level of 90%. Assume the cost of testing each subsystem is different, and limited samples are available to test for each subsystem. This information is given below.

Component	Cost/Per Sample	Available Samples
1	\$100	40
2	\$150	20
3	\$250	No Limit
4	\$900	100

Table 3- Cost and Available Samples for Each Subsystem

The total cost for the demonstration tests at the subsystem level should be controlled. The reliability allocation becomes

an optimization issue as:

$$\text{Minimize: } Cost(n_1, n_2, n_3, n_4) = 100n_1 + 150n_2 + 250n_3 + 900n_4$$

$$st: n_1 \leq 40, n_2 \leq 20, n_4 \leq 100$$

$$R_{sL} = Inv_Beta(0.1, a, b) \geq R_s^* = 0.95, 1 - CL_i = R_i^n$$

$$a = f_a(R_1, R_2, \dots, R_k), b = f_b(R_1, R_2, \dots, R_k)$$

If we set $CL_i = 0.9$ and use a zero-failure test for each subsystem, then the optimal solutions are:

Component	Test Cost	Test Samples	CL	Allocated Reliability R_i
1	\$3,600	36	0.9	0.9380
2	\$3,000	20	0.9	0.8913
3	\$26,750	107	0.9	0.9787
4	\$56,700	63	0.9	0.9641
Total Cost	\$90,050			

Table 4- Allocated Reliability Based on Cost

The allocated subsystem reliability at a confidence level of 90% will meet the system reliability requirement. At the same time, the above allocation minimizes the total test cost.

3.2 Risk Based Allocation

In certain regulated industry (medical device, aerospace, food, drug, etc.), safety is more important than cost. Risk critical components should have higher reliability than components that are less important. One way to quantify risk is:

$$Risk_i = (1 - R_i) \times Cost_i$$

where $Cost_i$ is the cost of a failure of subsystem i , representing the severity of failure consequence. For the system in Figure 1, assume the costs for each failure event are:

Component	Cost of Event
1	\$100
2	\$200
{1, 2}	\$2,000
3	\$3,000
4	\$5,000

Table 5- Cost of Each Failure Event

{1, 2} means both component 1 and 2 failed during a mission. The reliability allocation becomes an optimization issue as:

$$\text{Minimize: } Risk = 100(1 - R_1) + 200(1 - R_2) + 3000(1 - R_3) + 5000(1 - R_4) + 2000(R_1 + R_2 - R_1 R_2)$$

$$st: n_i \leq 100 (i=1,2,3,4)$$

$$R_{sL} = Inv_Beta(0.1, a, b) = 0.95, 1 - CL_i = R_i^n$$

$$a = f_a(R_1, R_2, \dots, R_k), b = f_b(R_1, R_2, \dots, R_k),$$

Constraints of sample size ($n_i \leq 100$) are added to the optimization problem and the calculated system reliability is set to the target of 0.95. Without these constraints, the allocated reliability for each component will be 1, which gives the lowest risk of 0. The optimal solution is:

Component	Test Samples	CL	Allocated Reliability R_i	Risk Cost
1	48	0.90	0.9532	\$4.68
2	99	0.90	0.9770	\$4.60
3	63	0.90	0.9641	\$107.67
4	99	0.90	0.9770	\$114.95
Both 1 and 2 Fail				\$2
Total Risk Cost				\$234

Table 6- Allocated Reliability Based on Risk

There are many different ways to define risk. For medical devices, a semi-quantitative method is given in BS EN ISO14971:2012. Different industries can easily apply the proposed reliability allocation method by defining their own objective functions.

3.3 Further Discussion

If one uses the allocated component reliability at a confidence level of 90% in Table 4 or 6 to calculate the system reliability, the result can be lower than the required reliability. For example using Table 6, we get:

$$R_s = (R_1 + R_1 - R_1 R_2) R_3 R_4 = 0.9409$$

which is lower than the required reliability of 0.95 at 90%. Is this right? To understand this, we need to look more deeply into the function of random variables. The system reliability is calculated from independent component reliabilities which are random variables. Define X_1 and X_2 as independent standard normal random variables and:

$$X = X_1 + X_2$$

We know the 90th percentile of X_1 and X_2 is 1.2816. X is also a normal distribution with mean of 0 and standard deviation of $\sqrt{2}$, so its 90th percentile is 1.8124. Clearly, $1.8124 \neq 1.2816 + 1.2816 = 2.5631$. Therefore, the sum of the percentiles of random variables is not the same as the percentile of the sum of random variables. For a system with a serial reliability configuration, we know:

$$R_s = \prod_{i=1}^k R_i \Rightarrow \ln(R_s) = \sum_{i=1}^k \ln(R_i)$$

The logarithm of the system reliability is the sum of the logarithm of the component reliabilities. Therefore, when we apply the traditional equal reliability allocation, the allocated reliability R_i does not have the same confidence level as the required system reliability. This is proved by the result in Table 2.

4 CONCLUSIONS

In this paper, we proposed a method for allocating a system reliability requirement with a given confidence level. The proposed method formulizes the allocation problem as an optimization issue. It is a general method and can be applied to systems with serial, parallel, and complex reliability configurations. All the constraints such as the cost, time, and other factors can be easily handled by the proposed method. Two examples are included in this paper to show how the

method can be applied.

REFERENCES

1. C. Ebeling, *An Introduction to Reliability and Maintainability Engineering*, McGraw-Hill, 1997.
2. W. Meeker, L. Escobar, *Statistical Methods for Reliability Data*, John Wiley & Sons, 1998.
3. A. Mettas, "Reliability Allocation and Optimization for Complex Systems," *Proc. Ann. Reliability & Maintainability Symp.*, (Jan.) 2000.
4. H. Guo, T. Jin, and A. Mettas, "Designing Reliability Demonstration Tests for One-Shot Systems Under Zero Component Failures," *IEEE Transactions on Reliability*, vol. 460, no. 1, 2011, pp. 286-294.
5. E. V. Huntington, "Frequency Distribution of Product and Sum," *Ann. Math. Statist.* vol. 10, 1939, pp. 409-421.
6. W. E., Thompson and R. D. Haynes, "On the Reliability, Availability, and Bayes Confidence Intervals for Multicomponent Systems," *Naval Research Logistics Quarterly*, vol. 27, 1980, pp. 345-358.
7. D.W. Coit, "System-reliability Confidence-Intervals for Complex-systems with Estimated Component-Reliability," *IEEE Transactions on Reliability*, vol. 46, no. 4, 1997, pp. 487-493.

BIOGRAPHIES

Huirui Guo
ReliaSoft Corporation
1450 S. Eastside Loop
Tucson, AZ, 85710

e-mail: Harry.Guo@ReliaSoft.com

Dr. Guo is the Director of the Theoretical Development Department at ReliaSoft Corporation. His research and publications cover reliability engineering, quality engineering, and applied statistics. In addition to research, he is also part of the training and consulting arm and has been involved in various projects from the automobile, medical device, oil and gas, and aerospace industries. He is a certified reliability professional (CRP), ASQ certified CQE and CRE. He is a member of IIE, SRE, and ASQ.

Mingxiao Jiang
Medtronic, Inc.
7000 Central Ave NE
Minneapolis, MN 55432, USA

e-mail: Mingxiao.jiang@medtronic.com

Dr. Jiang received his B.S. in Engineering Mechanics from Zhejiang University, M.S. in Reliability Engineering from the University of Arizona, and Ph.D. in Mechanics of Materials from the Georgia Institute of Technology. He is currently working as a Senior Principal Reliability Engineer at Medtronic Neuromodulation. He also serves as an Adjunct Professor at Zhejiang University. He is a CRE, a member of SRE, a senior member of ASQ and a senior member of IEEE. He has 3 US patents and 30 publications in refereed journals

and conference proceedings.

Wendai Wang
Thoratec Corporation
6035 Stoneridge Drive
Pleasanton, CA, 94588, USA

e-mail: Wendai.wang@thoratec.com

Dr. Wang received his B.S. and M.S. from Shanghai Jiaotong University and his Ph.D. from the University of Arizona.

Currently he is a staff reliability engineer at Thoratec Corporation. Prior to Thoratec, he was a reliability technical leader/manager at GreenVolts, Applied Materials, GE, and Honeywell. He is the author of over 40 publications, and has successfully led many innovative Design for Reliability (DFR) projects. His work and research areas include DFR methodology, reliability planning, reliability testing, supplier quality and reliability, reliability assessment and qualification, system reliability / availability modeling and analysis, mechanical and electronics reliability, and reliability training.