Demonstrating Reliability Growth Requirements with Confidence

Larry H. Crow, Ph.D., Crow Reliability Resources

Key Words: Reliability Growth; Demonstration Test; Requirements; Confidence Bounds**;** Demonstrating Requirements

SUMMARY & CONCLUSIONS

Reliability growth testing is used to find reliability problems in the system design through testing so that effective corrective actions can be taken to improve the reliability. In addition, many systems are subjected to a follow-on reliability demonstration test and the results are typically compared to a reliability requirement. The expectation is that by conducting the reliability growth test the probability of passing the demonstration test is improved. While this is generally true for the original requirement, in many cases the criteria for passing the demonstration test is to demonstrate the requirement with confidence as a lower bound, for example 80%. This criteria, in effect, changes the original requirement and design goals, and if the lower confidence bound is based strictly on demonstration test data, this may dramatically lower the probability of passing the demonstration test. In order to pass the demonstration test with a high probability, say 80%, then the true design reliability needs to be much higher than the requirement. The higher the true design reliability needs to be the higher the associated engineering costs and the higher the risk of not passing the demonstration test. This paper develops a methodology that utilizes both the reliability growth test data and the demonstration test data in order to (1) demonstrate the requirement with confidence and (2) with sufficient reliability growth lower the true design reliability that is necessary to pass the demonstration test with a high probability. The focus is to reduce risks and costs. This paper is limited to a system with a continuous time to failure, and it is assumed that the Crow (AMSAA) model applies to the reliability growth results. The specific problem addressed is a recognized important issue, particularly in the Department of Defense. The objective of this paper is to provide a practical solution to this problem.

1 INTRODUCTION

In this paper we consider the real world situation where a system with continuous time to failure first undergoes a reliability growth test and is then subjected to a demonstration test in which the reliability is held fixed. During the reliability growth test reliability problems are found and corrective actions are incorporated into the system in order to increase the system reliability, or specifically, increase the Mean Time Between Failure (MTBF). If a failure occurs during the demonstration test the system is simply repaired and put back on test.

We assume that there is a system MTBF requirement and the focus of the demonstration test is to demonstrate this requirement with high statistical confidence. In this paper the intent is to keep all of the standard concepts in place in terms of the current Department of Defense (DoD) approach to the demonstration test objectives. Specifically, this paper does not discuss a Bayesian approach because that is not an option in the current DoD demonstration test methodology. The current trend in DoD for reliability demonstration tests is different from approaches taken in the past. Therefore, in order to make the problem addressed in this paper concrete some terms and concepts need to be reviewed.

A typical reliability demonstration test is governed by an Operating Characteristics (OC) curve. For a specified true MTBF the OC curve gives the probability of passing the demonstration test. Associated with the OC curve are consumer and producer risks. At a lower MTBF, $MTBF_C$, the consumer risk, R_C, is the probability that a system with a MTBF as low as $MTBF_C$ will pass the demonstration test. The probability R_C is the value on the OC curve at the MTBF value $MTBF_C$. At a higher MTBF, $MTBF_P$, the producer risk, R_P, is the probability that a system with a MTBF as high as $MTBF_P$ will not pass the demonstration test. The probability R_P is 1 minus the value on the OC curve at the MTBF value $MTBF_P$. An organization that is paying to have a product or system developed may be considered both a consumer and a producer. If the role of producer is taken then the producer's MTBF, $MTBF_P$ is set at the requirement MTBF, $MTBF_{Req}$. The consumer MTBF is then set at a lower value so that there is a low probability the organization will accept a system this "bad." For this type of demonstration test approach the inputs (1) $MTBF_{Req} = MTBF_P$ (2) R_P (3) $MTBF_C$ (4) R_C, are given and the demonstration test time T_D and the maximum allowable number of failures N are determined so that the desired OC curve probabilities R_C and 1- R_P are obtained at $MTBF_C$ and $MTBF_P$, respectively. With this demonstration test approach, the test time T_D is a dependent parameter.

In many cases the risks R_P and R_C are set equal, for example, 20%, but this is not necessary. In addition, there is an important relationship between demonstration tests and a lower confidence bound on $MTBF_C$. If the demonstration test is conducted and the number of observed failures is less than the maximum allowable, then the lower 1- R_C confidence bound on the true MTBF is at least $MTBF_C$. This result is key to the methodology presented in this paper. However, in the case discussed above the $MTBF_C$ is not the requirement.

The intent of a demonstration test in which the producer MTBF, $MTBF_P$, is set at the requirement $MTBF_{Req}$, as noted above, is not to demonstrate the requirement with high confidence but to test the hypotheses that the true MTBF is

978-1-4577-1851-9/12/$26.00 ©2012 IEEE

equal to the producer's MTBF or the requirement. However, of particular interest is the length T_D of the demonstration test and associated costs. In the past this demonstration test approach has been common, particularly in the DoD.

In 1977, Crow, as Chief of the Army Materiel Systems Analysis Activity (AMSAA) Reliability Methodology Office, developed confidence interval procedures on the MTBF achieved at the end of reliability growth testing based on the Crow (AMSAA) model. (See Ref. 3). This involved developing, in Ref. 3, for the first the distribution of the number of failures during the growth test, as a function of the achieved MTBF and test time. This distribution is also in Ref. 4 and Ref. 6. Crow observed that this distribution could be used to combine the number of reliability growth failures with the demonstration test failures and suggested to G. Miller, a member of this office, that he utilize this result to reduce the demonstration test time, T_D. This was done in Ref. 5, for the demonstration test situation described above where the MTBF requirement is set at the producer's MTBF value $MTBF_P$. The stated objective of Miller was to reduce the length of the demonstration test while controlling the overall risks. However, if the system passed the demonstration test, the lower 1- R_C (e.g. 80%) confidence bound is the consumer MTBFC, not the MTBF requirement $MTBF_P$.

Currently, a widely used approach in DoD demonstration tests is not the one described above but one that is focused on demonstrating the requirement with high confidence. In this paper we will use the result of Crow (1977) to again combine reliability growth data and demonstration test data, but we will address a different demonstration test objective than was discussed in Ref. 5.

If we want a demonstration test so that we demonstrate the requirement with high confidence, then, as noted above, the MTBF requirement must be set at the consumer MTBF value $MTBF_C$. That is, $MTBF_{Req} = MTBF_C$. This means that the actual design MTBF (i.e. $MTBF_P$) that is necessary to pass the demonstration test with high probability must be well above the requirement. For example, consider the real world situation where the reliability requirement is 105 hours MTBF, and the duration of the reliability demonstration test is 1000 hours. In order to demonstrate the 105 hour requirement at 80% confidence the system must have 6 or fewer failures in 1000 hours. (See eq. 4). That is, by definition, if the system has exactly an MTBF of 105 then there is only a 20%, or less, probability that the system will have 6 or fewer failures and pass the demonstration test. Therefore, to have an 80 % probability of having 6 or fewer failures in the 1000 test the system must have a MTBF of 211 or higher. (See eq. 5). This means that the target MTBF for reliability growth planning is 211 hours, not 105 hours.

The situation above has strong implications for a design solution for the system. For example, in reliability growth testing it is not possible to mature the system MTBF above the system's design growth potential. It is typical to expect that the design growth potential be no more that 50% above the requirement as a cost effective design solution. That is, the target requirement of 105 may require a design solution with a growth potential 50% above the requirement or 158 hours MTBF. The 80 % confidence constraint, however, increases the target MTBF by two times the requirement, or 211 hours MTBF and may then require a higher growth potential design solution, say, 50% higher, or 316 hours MTBF. This is three times the original 105 hour MTBF requirement.

The design goal of 211 is determined by the requirement and available test time during the demonstration test, and may not be supported by existing technology, budget, etc. Therefore, the higher the design goal, or producer's MTBF the higher the risks of failing the demonstration test, even if the requirement has been exceeded.

The objective of this paper is to use the reliability growth data together with the demonstration test data to lower the producer's MTBF, $MTBF_P$, and still demonstrate the requirement with the stated confidence. In the example, we want to reduce, if possible, the $MTBF_P$ from 211 to a lower value. With the methodology presented in this paper there is still an 80% probability of passing the demonstration test at this new lower $MTBF_P$ value and we will still keep the consumer's MTBF and risk at the requirement. In our example, there will still be a 20% probability of passing the demonstration test if the system MTBF equals the requirement of 100 hours. Importantly, under this methodology, if the demonstration test is passed, then the requirement has been demonstrated with the same stated statistical confidence. With our example this means that if the demonstration test is passed then the requirement of 105 hours is demonstrated with 80 % confidence, the objective.

The author is not aware of previous published work that addresses this specific problem.

2 *DEMONSTRATION TEST CONSUMER AND PRODUCER RISKS*

Assuming exponential failure times with mean θ during the demonstration test the total number of failures during the demonstration test of length T_D is exactly distributed according to the Poisson distribution with mean $\phi = T_D/\theta$. If the system is not exponential then the Poisson distribution is considered an approximation and is commonly used. The probability that the number of failures is exactly n, n = 0, 1, 2,…, is given by the Poisson as

$$\Pr[N = n \mid \phi = \phi_0] = \frac{(\phi_0)^n e^{-\phi_0}}{n!} \quad (1)$$

The probability that the number of failures is less than or equal to n (where *n*=0, 1, ….) is given by:

$$\Pr[N \le n \mid \phi = \phi_0] = \sum_{i=0}^{n} \frac{(\phi_0)^i e^{-\phi_0}}{i!} \quad (2)$$

At the requirement, the Poison mean is:

$$\phi_R = \left(T_D / MTBF_{Req} \right) \quad (3)$$

In our example, the demonstration test is 1000 hours and the requirement is 105 hours. At the requirement, the Poisson mean in this example is:

$$\phi_R = \left(1000/105\right) = 9.52$$

How many failures are allowed in a 1000 hour test to demonstrate the 105 hour requirement with 80% confidence? We use the Poisson distribution to find the maximum n_0 such that:

$$\Pr[N \le n_0 \mid \phi = \phi_R] = \sum_{i=0}^{n_0} \frac{(\phi_R)^i e^{-\phi_R}}{i!} \le 0.20 \quad (4)$$

In this example $n_0 = 6$. If the number of failures in the 1000 hour demonstration test is 6 or less then the 105 hour requirement is demonstrated with at least 80 % confidence. However, there is a 20 % or less probability of 6 or fewer failures if the system MTBF is exactly equal to the requirement of 105 hour MTBF. How large does the MTBF need to be so that there is an 80 % probability of 6 or fewer failures in the 1000 demonstration test? We find the MTBF θ in $\phi = T_D/\theta$ such that:

$$\Pr[N \le n_0 \mid \phi] \le \sum_{i=0}^{n_0} \frac{(\phi)^i e^{-\phi}}{i!} = 0.80 \quad (5)$$

This gives an MTBF = 211 for t_d = 1000 hours. at an MTBF of 211 there is an 80% probability of passing the demonstration test and a 20 % probability of failing the test. This means that if we use only the 1000 hour demonstration test data we need to attain a system with a high MTBF of 211 hours at the end of the reliability growth test to have an 80% probability of passing the demonstration test and demonstrating 105 hours MTBF with confidence. This is the issue addresses in this paper.

3 CROW(AMSAA) RELIABILITY GROWTH MODEL

The Duane postulate, [1], for reliability growth during test-fix-test development testing stated that the instantaneous system failure rate at cumulative test time t is $r(t) = \lambda\beta t^{\beta-1}$, where $0 < \lambda$ and $0 < \beta$ are parameters.

Crow [2] modeled the Duane postulate stochastically as a non-homogeneous Poisson process (NHPP) with intensity function

$$r(t) = \lambda\beta t^{\beta-1} \quad (6)$$

thus, allowing for statistical procedures based on this process for reliability growth analyses. The parameter λ is referred to as the scale parameter and β is the shape, or growth, parameter. For $\beta = 1$, there is no reliability growth. For $\beta < 1$, there is positive reliability growth. That is, the system reliability is improving due to corrective actions. In addition, for $\beta > 1$, there is negative reliability growth.

Under the Crow (AMSAA) model, the achieved or demonstrated failure intensity at time T, the end of the test, is given by

$$r(T) = \lambda\beta T^{\beta-1} \quad (7)$$

and the achieved MTBF at time T is given by

$$M(T) = 1/\lambda\beta T^{\beta-1}. \quad (8)$$

Suppose a development testing program begins at time 0 and is conducted until time T and stopped. Let N be the total number of failures recorded and let $0 < X_1 < X_2 < \ldots X_N \le T$ denote the N successive failure times on a cumulative time scale. We assume that the Crow (AMSAA) model assumptions apply to this set of data. Under the model the maximum likelihood estimates (MLEs) for λ and β are:

$$\hat{\lambda} = \frac{N}{T^{\hat{\beta}}}, \qquad \hat{\beta} = \frac{N}{\sum_{i=1}^{N} \ln(\frac{T}{X_i})}. \quad (9)$$

4 METHODOLOGY TO COMBINE RELIABILITY GROWTH TEST AND DEMONSTRATION TEST

In eqs. 4 and 5 the demonstration test OC curve probabilities are determined by the Poisson distribution with mean $\phi_D = T_D/\theta_D$ where θ_D is the system MTBF during the demonstration test. The Poisson calculation is based on the total number of failures observed during the demonstration test. In this section we will use the result of Crow, Ref. 3, in which he showed that conditioned on the calculated value of W at the end of the reliability growth test, given by eq. 11, the distribution of the total number of failures during the reliability growth test is governed by the parameter $\phi_{RG} = T_{RG}/M(T_{RG})$, where $M(T_{RG})$ is the final MTBF, (see eq. 8), at the end of the reliability growth test. This conditional distribution is also in Ref. 4 and Ref. 6. This desired distribution is given in eq. 20 below. We then set the MTBF at the end of the reliability growth test as the MTBF during the demonstration test. That is, the Poisson distribution mean is $\phi_D = T_D/M(T_{RG})$. As noted below, we can also account for a conversion factor or degradation factor in relating the final MTBF in the reliability growth test to the MTBF in the demonstration test. Because we know T_{RG} and T_D the conditional distribution of the number of failures in the reliability growth test and the number of failures in the demonstration test have the same parameter $M(T_{RG})$. For any specified value of $M(T)$ we use eq. 20, conditioned on W, to combined the test data in the reliability growth test and the demonstration test. Our new OC curve is now based on the total number of failures in both tests and is conditional on the results of the reliability growth test. We use this result to lower, if possible, the producer's MTBF, and increase the probability of passing the demonstration test if the requirement is exceeded.

Based on eq. 9, the Crow (AMSAA) model maximum likelihood estimate of β is

$$\hat{\beta} = \frac{N}{W} \quad (10)$$

where

$$W = \sum_{i=1}^{N} \ln(\frac{T_{RG}}{X_i}). \quad (11)$$

Under the Crow (AMSAA) mode, for N = n, the probability density function (p.d.f.) of $X_1, \ldots, X_n$ is

$$f(x_1, x_2, \ldots, x_n, \lambda, \beta) = \lambda^n \beta^n e^{-\lambda T^\beta} \prod_{i=1}^{n} x_i^{\beta-1} \quad (12)$$

Under this model the unconditional number of failures N during reliability growth has the Poisson distribution with mean $\theta = \lambda T^{\beta}$. What Crow, Ref. 3, derived was the p.d.f. of N given the results of the reliability growth test, i.e. specifically given the value of W in eq. 11. This result is given in eq. 16 below.

To see this result (eq. 16) we note from Crow, Ref. 3 that the conditional p.d.f. of $X_1,...,X_n$) given $N = n$ is

$$f(x_1, x_2,...,x_n, \beta \big| N = n) = n! \prod_{i=1}^{n} \beta x_i^{\beta-1} \big/ T^{\beta}. \qquad (13)$$

That is, the ordered times $x_1, x_2,...,x_n$, conditioned on N = n, are distributed as order statistics for a sample of size n from a distribution with p.d.f. $\beta x^{\beta-1} \big/ T^{\beta}$. Hence, the p.d.f. of W, (eq. 11) given $N = n \geq 1$, is

$$q(w,\beta \mid N = n) = \beta^n w^{n-1} e^{-\beta w} \Big/ (n-1)! \qquad (14)$$

$w > 0,\ n = 1,2,...$. Therefore, as shown in Crow [3], the joint p.d.f. of (N, W), given W>0, is

$$p(n,w,\phi_{RG},\beta) = \left(\phi_{RG}^n w^{n-1} e^{-\beta w - \phi_{RG}/\beta} \Big/ n!(n-1)! \right) (1 - e^{-\phi_{RG}/\beta}) \quad (15)$$

$w > 0,\ n = 1,2,...$, where $\phi_{RG} = T_{RG} \big/ M(T_{RG})$, and $M(T_{RG})$ is the achieved reliability growth MTBF, given by eq.8, at the end of the reliability growth test.

What we want is the conditional p.d.f. of the total number of failures in the reliability growth test. From eq. 15 Crow, [3] p.16, eq. 5.4, determined the p.d.f. of N given $W = w > 0$. The conditional p.d.f. is given by:

$$p(n,\phi \mid W = w) \frac{\left(\phi_{RG}^n w^{n-1} \Big/ n!(n-1)! \right)}{\sum_{j=1}^{\infty} \left(\phi_{RG}^j w^{j-1} \Big/ j!(j-1)! \right)} \qquad (16)$$

Eq. 16 is used to determine the conditional distribution of the combined total number of failures in the reliability growth test and the demonstration test. This distribution is given in eq. 20.

It is generally recommended that reliability growth tests be conducted under operational conditions, essentially the same as the demonstration test. This means that the MTBF, $M(T)$, at the end of the reliability growth test should be comparable to a reasonable degree to the MTBF during the demonstration test. Under these conditions the mean is $\phi_D = T_D / M(T)$ in the Poisson distribution in Section 2 for the number of failures in the demonstration test, and the number of failures in the reliability growth test, conditioned on W =w has the parameter ϕ_{RG} where $\phi_{RG} = T_{RG} / M(T)$ in eq. 16. If a conversion factor C_F is assumed, for example, for MTBF degradation going from the reliability growth test to the demonstration test the calculations are the same with the $\phi_{RG} = T_{RG} / (C_F) \cdot M(T_{RG})$.

What this means is that conditioned on W=w, the distribution of total number of failures in the combined reliability growth test and the demonstrated test have the same governing parameter $M(T)$.

The consumer and producer risks during the demonstration test can now be determined as in Section 2, but with a new distribution instead of the Poisson. In our example the MTBF = 211 is necessary in order to have an 80% Poisson probability of passing the demonstration test. The objective of this paper is to use the reliability growth data together with the demonstration test data to lower this MTBF while keeping the consumer's risk fixed at 20% at the requirement MTBF = 105. That is, the requirement of 105 hours will be demonstrated with 80% confidence.

With this methodology the reliability growth test is first conducted and the value W = w is calculated using eq. 11. Conditioned on the value of w and the reliability growth test time T_{RG} , eq. 16 calculates the probability that the total number of failures in the reliability growth test, N_{RG} , is equal to n_{RG} , $n_{RG} = 1,2,....$ for the test time T_{RG} and for any value of $M(T_{RG})$. Note that T_{RG} and $M(T_{RG})$ gives $\phi_{RG} = T_{RG} \big/ M(T_{RG})$ in eq. 16.

We want to utilize information in both the reliability growth test and the demonstration test for demonstrating the MTBF requirement with confidence. We are, therefore, interested in the total number of failures, $N^* = N_{RG} + N_D$, for both tests.

We use eq.1 and eq. 16, to calculate the probability

$$\Pr(N^* \leq n^* \big| w, MTBF = M(T)). \qquad (17)$$

To do this we note that:

$$\begin{aligned} &\Pr(N^* \leq n^* \big| w, MTBF = M(T)) \\ &\quad = \Pr(N^* = 1 \big| w, MTBF = M(T)) \\ &\quad\quad + \Pr(N^* = 2 \big| w, MTBF = M(T)) \\ &\quad\quad + \cdots \\ &\quad\quad + \Pr(N^* = n^* \big| w, MTBF = M(T)) \end{aligned}$$

and

$$\begin{aligned} &\Pr(N^* = j \big| w, MTBF = M(T)) = \Pr(n_{RG} = 1) \cdot \Pr(n_D = j-1) \\ &\quad + \Pr(n_{RG} = 2) \cdot \Pr b(n_D = j-2) + \cdots + = \Pr(n_{RG} = j) \cdot \Pr(n_D = 0) \end{aligned}$$

The $\Pr ob(n_{RG} = j)$ is given by eq. 16, and the $\Pr ob(n_D = i)$ is given by eq. 2, with respective parameters

$$\phi_{RG} = T_{RG} \big/ M(T_{RG}),\ \phi_D = T_D / M(T_{RG}).$$

For growth we must have at least one failure during the reliability growth test. That is, we assume $n_{RG} \geq 1$.

In general,

$$\Pr(N^* = n^* \big| w, MTBF = M(T)) = \sum_{j=1}^{n^*} \frac{\left[\phi_{RG}^j w^{j-1} \big/ j!(j-1)! \right]}{F(\phi_{RG}, w)} \cdot \frac{\left[\phi_D \right]^{n^*-j} e^{-\phi_D}}{(n^*-j)!} \qquad (18)$$

where

$$F(\phi_{RG}, w) = \sum_{j=1}^{\infty} \left(\phi_{RG}^j w^{j-1} \big/ j!(j-1)! \right). \qquad (19)$$

Therefore, the distribution of interest is given by

$$\Pr(N^* \leq n^* \big| w, MTBF) = \sum_{j=1}^{n} \sum_{i=1}^{j} \frac{\left[\phi_{RG}^i w^{i-1} \big/ i!(i-1)! \right]}{F(\phi_{RG}, w)} \cdot \frac{\left[\phi_D \right]^{j-i} e^{-\phi_D}}{(j-i)!}. \qquad (20)$$

The objective of this paper is to use the information in the total number of failures in the reliability growth test coupled with the total number of failures in the demonstration test to (1) demonstrate the requirement with confidence, and (2) if possible, lower the producer $MTBF_P$ in order to increase the probability of passing the demonstration test if the design MTBF exceeds the requirement. This methodology is illustrated in the following 5 steps:

Step (1). First set $M(T_{RG})$ equal to the requirement, $MTBF_{Reg}$. That is, we set $\phi_{RG} = T_{RG}/MTBF_{Reg}$, $\phi_D = T_D / MTBF_{Reg}$.

Step (2). After the reliability growth test is completed with n_{RG} failures we use the reliability growth test data and eq. 11 to calculate $w = \sum_{i=1}^{n} \ln(\frac{T_{RG}}{X_i}) = \frac{n_{RG}}{\hat{\beta}}$.

Step (3). To use the reliability growth data to design the demonstration test of fixed length T_D in order to demonstrate the requirement at 80 % confidence, we next use inputs from Step 1 and Step 2 to find the maximum n^*, denoted by n_0^*, such that

$$\sum_{j=1}^{n_0^*}\sum_{i=1}^{j} \frac{\left[\phi_{RG}^i w^{i-1}/i!(i-1)!\right]}{F(\phi_{RG}, w)} \cdot \frac{\left[\phi_D\right]^{j-i} e^{-\phi_D}}{(j-i)!} \le 0.20, \quad (21)$$

where $\phi_{RG} = T_{RG}/MTBF_{Req}$ and $\phi_D = T_D/MTBF_{Req}$.

This sets the consumer's risk, e.g. 20%, at the requirement. Conditioned on w, if the number of failures in the reliability growth test and the demonstration test is a total of n_0^* or less, then the requirement is demonstrated, (in this example), with 80% confidence.

Step (4). To determine the new producer MTBF we set $\phi_{RG} = T_{RG}/MTBF_P$, and $\phi_D = T_D/MTBF_P$. For the n_0^* determined in Step (3), we then find the value of $MTBF_P$ such that

$$\sum_{j=1}^{n_0^*}\sum_{i=1}^{j} \frac{\left[\phi_{RG}^i w^{i-1}/i!(i-1)!\right]}{F(\phi_{RG}, w)} \cdot \frac{\left[\phi_D\right]^{j-i} e^{-\phi_D}}{(j-i)!} = 0.80. \quad (22)$$

If the final MTBF at the end of the reliability growth test is sufficiently high, then the new $MTBF_P$ calculated from eq. (22) will be lower than the $MTBF_P$ calculated from eq. 5. In our example the MTBF calculated from eq. 5 based only on the 1000 demonstration test was 211. A conversion factor will modify the calculations accordingly.

Step (5). The number of failures observed during the reliability growth is n_{RG}. Next conduct the demonstration test and observe the total number of failures n_D. The total allowed number of failures during the demonstration test in order to demonstrate the requirement with confidence is $n_D = n_0^* - n_{RG}$. If $n_D \le n_0^* - n_{RG}$ then the demonstration test is passed and the requirement is demonstrated with confidence. If $n_D > n_0^* - n_{RG}$ then the demonstration test is failed.

General guidelines for applying this methodology is for reliability improvement to occur during the reliability growth test, that is, $\hat{\beta} < 1$ and the reliability growth MTBF estimate

$$\hat{M}(T_{RG}) = \frac{T_{RG}}{n_{RG} \cdot \hat{\beta}} \quad (23)$$

is greater than the requirement M_R.

5 NUMERICAL EXAMPLE

Consider again the example discussed in Section 2 where the MTBF requirement $MTBF_{Req} = 105$ and this requirement is to be demonstrated with 80% with a demonstration test of $T_D = 1000$ hours. As calculated in Section 2, the maximum allowable number of failures during the demonstration test in order to demonstrate the 105 hour requirement at 80 % confidence is 6. This requires a demonstrated MTBF at least 166.6 during the reliability growth test, and, importantly, requires an actual system MTBF of 211 in order to pass this test with an 80 % probability. That is the key issue addressed with the methodology discussed in this paper. The intent is to use additional information so that the high design MTBF of 211 may be lowered. This will be the case if the MTBF demonstrated during the reliability growth test is sufficiently high. A design MTBF as high as MTBF of 211 may not necessary. Suppose there is also a reliability growth test of 4300 hours. We then follow Step1 through 5 below.

Step (1). Calculate $\phi_{RG} = (4300/105) = 40.95$, $\phi_D = (1000/105) = 9.52$.

Step (2). During the reliability growth test time of $T_{RG} = 4300$ hours there are 40 failures. The observed failure times are: 35.9,88.1,148.5,215.1, 286.6, 362.4, 442.0 524.9, 610.8, 699.5,790.8,884.5,980.5,1078.6,1178.8,1280.9,1384.8,1490.0,1597.9,1707.0,1817.6,1929.8,2043.4,2158.4,2274.9, 2392.6, 2511.7,2632.1,2753.7,2876.5, 3000.5 3125.6 ,3251.9, 3379.3, 3507.7, 3637.2, 3767.8, 3899.3, 4031.9, 4165.4.

Based on eq. 11 we calculate

$$w = \sum_{i=1}^{40} \ln(4300/X_i) = 49.2. \quad (24)$$

We also calculate

$$\hat{\beta} = (40/w) = 0.813. \quad (25)$$

The Crow (AMSAA) model MTBF estimate at the end of the reliability growth test is

$$\hat{M}(T_{RG}) = \frac{1}{\hat{\lambda}\hat{\beta}T_{RG}^{\hat{\beta}-1}} = \frac{T_{RG}}{n_{RG} \cdot \hat{\beta}} = 132.2 \text{ hours.} \quad (26)$$

Step (3). For $w = 49.2$, $\phi_{RG} = 40.95$, $\phi_D = 9.52$, we use eq. 21 to find the maximum allowable failures n_0^*. This gives

$$n_0^* = 49. \quad (27)$$

That is, conditioned on w = 49.2 , if there are a total of 49 or fewer failures combined in the reliability growth test and the demonstration test, then the MTBF requirement of 105 hours is demonstrated with 80 % confidence. (Assuming no conversion factor). As a standalone demonstration test we could allow only 6 failures in the 1000 hour demonstration test and, therefore, must have a minimum average MTBF over the 1000 hour demonstration test of 166.6.

Treated as a standalone test, the system developer would need an actual MTBF of 211 in order to have 6 or fewer failures in the demonstration test, and, therefore demonstrate the 105 MTBF requirement with 80 % confidence. This means that if the requirement is exceeded, but not as high as 211, then there is less than an 80 % probability that the system will pass the demonstration test. By using the reliability growth test results we can generally lower the MTBF required to demonstrate the MTBF requirement with confidence.

Step (4). To determine the new producer MTBF we set $\phi_{RG} = 4300/MTBF_P$, and $\phi_D = 1000/MTBF_P$. For $n_0^* = 49$, w = 49.2, we then find the value of $MTBF_P$ such that

$$\sum_{j=1}^{n_0^*}\sum_{i=1}^{j}\frac{\left[\phi_{RG}^{i} w^{i-1}/i!(i-1)!\right]}{F(\phi_{RG},w)}\cdot\frac{\left[\phi_D\right]^{j-i}e^{-\phi_D}}{(j-i)!}=0.80. \quad (28)$$

This gives $MTBF_P = 153.8$. This says that going into the demonstration test if the true system MTBF is at least 153.8, there is an 80% or greater probability of passing the demonstration test and demonstrating the requirement with 80 % confidence.

Step (5). Because we have already seen 40 failures in the reliability growth test we can allow up to 9 failures in the 1000 hour demonstration test and still demonstrate the 105 hour requirement with 80 % confidence. This is an average MTBF over the 1000 hour demonstration test of 111.1.This is considerably less than the 166.6 hour average MTBF necessary if we consider only the demonstration test data.

The calculations in this example were done using Microsoft Excel. The denominator in eq. 20 converged very quickly. If a computer program is used then we note from Refs.3, 4, 6, that eq. 20 may be also be written as

$$\sum_{j=1}^{n}\sum_{i=1}^{j}\frac{\left(w\cdot\phi_{RG}\right)^{i-1}}{i!(i-1)!\cdot I_1\left(2\sqrt{w\cdot\phi_{RG}}\right)}\cdot\frac{\left[\phi_D\right]^{j-i}e^{-\phi_D}}{(j-i)!} \quad (29)$$

where $I_1(\cdot)$ is the modified Bessel function of order 1.

REFERENCES

1. J. T. Duane, Learning Curve Approach to Reliability Monitoring, IEEE Transactions on Aerospace. Vol. 2, 1964, PP.563-566.
2. L. H. Crow, Reliability Analysis for Complex, Repairable Systems, in Reliability and Biometry, ed. by F. Proschan and R.J. Serfing, pp.379-410, 1974, Philadelphia, SIAM.
3. L.H. Crow, Confidence Interval Procedures For Reliability Growth Analysis, AMSAA TR No. 197, June 1977. U. S. Army Materiel Systems Analysis Activity, Aberdeen Proving Ground, MD.
4. Bain, L.J. and M. Engelhard, Inferences on the Parameters and Current Reliability For a Time Truncated Weibull Process, Technometrics 1980, Vol. 22.
5. G. W. Miller, An Extension To The Weibull Process Model, AMSAA TR No. 341, November 1981. U. S. Army Materiel Systems Analysis Activity, Aberdeen Proving Ground, MD.
6. L. H. Crow, Confidence Interval Procedures for the Weibull Process with Applications to Reliability Growth, Technometrics, 1982, Vol. 24.

BIOGRAPHY

Larry H. Crow, Ph. D.
109 Clifts Cove Blvd.
Madison, AL 35758 U. S. A.

Internet (e-mail) Crowrel@knology.net

Dr. Larry H. Crow is president of CRR. Previously Dr. Crow was VP, Reliability & Sustainment Programs, at ALION Science and Technology, Huntsville, AL. From 1985 to 2000, Dr. Crow was Director, Reliability, at General Dynamics ATS -- formally Bell Labs ATS. From 1971-1985, Dr. Crow was chief of the Reliability Methodology Office at the US Army Materiel Systems Analysis Activity (AMSAA). He developed the Crow (AMSAA) reliability growth model, which has been incorporated into US DoD handbooks, and national & international standards. He chaired the committee to develop Mil-Hdbk-189, 1981, *Reliability Growth Management,* and is the principal author of that document. Dr. Crow is a Fellow of the American Statistical Association, and the Institute of Environmental Sciences and Technology. He is a Florida State University Alumni Association Distinguished Alumnus and the recipient of the FSU "Grad Made Good" Award for the Year 2000, the highest honor given to a graduate by Florida State University. Dr. Crow is the recipient of the 2011 Council of Logistics Engineering Professionals' Blanchard-Langford Award.