Design of Reliability Demonstration Testing for Repairable Systems

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SUMMARY & CONCLUSIONS

Reliability Demonstration Testing (RDT) for non-repairable systems has been successfully implemented in many industries, such as microelectronics, aerospace and healthcare. In designing RDTs, the famous beta-binomial formula is often applied to determine the necessary sample size so that the required reliability metric at a given confidence level can be demonstrated. However, many systems, such as cars and combine harvesters, are repairable systems. For repairable systems, RDT design methods have been developed mainly for Homogeneous Poisson Processes (HPP). This paper provides a new RDT design method applicable for both HPP and Non-Homogeneous Poisson Process (NHPP) cases. The result shows that the proposed method can effectively determine the required testing time and sample size of test units to demonstrate the required Mean Time Between Failures (MTBF). A comparison study is also conducted to illustrate the superior performance of the proposed method over some of the existing methods.

1 INTRODUCTION

RDT is a process that demonstrates whether a product has met a certain reliability requirement with a specific confidence. It is usually performed at the system level and is typically set up as a pass/fail test. Because of some similarities between RDTs for non-repairable systems and RDTs for repairable systems, we will briefly review the background theories for both types of systems first and then discuss the drawbacks of existing RDT methods for repairable systems.

1.1 RDT Design for Non-Repairable Systems

Design of RDTs for non-repairable systems has been extensively studied. In general, all of the existing design methods can be classified into two categories: (1) Test design based on number of failures [1-3] and (2) Test design based on failure time [4-6].

Test design based on the number of failures is the most commonly used RDT design method. It is also called a "binomial test" because of the use of the binomial equation:

$$1-CL=\sum_{i=0}^{r}\binom{n}{i}\left(1-R(T)\right)^{i}R(T)^{n-i} \tag{1}$$

where n is the number of test units, r is the number of failures, $R(T)$ is the lower bound on the reliability at time T and CL is the given confidence level. From Eqn. (1), one can see that this design method does not assume a specific failure time distribution. If any three out of the four variables are given, the remaining variable can be determined. Due to the nonparametric nature, this method has been widely used.

An alternative parametric RDT design method is based on a given failure time distribution and the required lower confidence bound on a specific failure percentile. Such a time-based test design method was first proposed by Lawless [4] and then extended by McKane, et al. [5]. For example, a manufacturer is required to demonstrate that a product's reliability at a given time t is higher than a stated reliability target. Suppose n units are tested, the test ends at time T and r failures are observed. The observed failure times and censoring times can be used to fit a failure time distribution. Once the distribution is obtained, the lower confidence bound on the reliability at any given time t can be obtained. If the lower bound is higher than the stated reliability target, the product passes the demonstration test. Since failure times are required for fitting a distribution. If no failures are observed, it is difficult, if not impossible, to estimate the model parameters. To deal with such cases, special methods have been developed. For example, for a Weibull distribution, when there are no failures, the scale parameter η can still be estimated at a given confidence level provided that the value of the shape parameter β is known. Specifically, the lower bound of η can be expressed as:

$$\eta_L=\left[2\sum_{i=1}^{n}t_i^{\beta}\Big/\chi^2_{CL,2}\right]^{1/\beta} \tag{2}$$

where t_i is the total test time for unit i and $\chi^2_{CL,2}$ is the left tail CL percentile of a Chi-Square distribution with 2 degrees of freedom. As a result, the lower confidence bound on reliability at time t can be expressed as:

$$R(t)=\exp\left[-\left(t/\eta_L\right)^{\beta}\right] \tag{3}$$

For zero failure cases, Eqn. (3) gives exactly the same result as Eqn. (1) (*i.e.* r = 0 in Eqn. (1)). This can be mathematically proven using the relationship between Eqn. (1) and the Chi-Square distribution. On the other hand, if failures are observed during the test, the two test design methods will give different results. Moreover, Eqn. (2) cannot be applied directly in such cases because the test time t_i (either failure time or censoring time) is unknown before the test is performed. To use the test design method based on failure time, the expected Fisher information of each test unit needs to be calculated. The information will be used to get the expected Fisher matrix bound for η and reliability [5].

1.2 RDT Design for Repairable Systems

Although repairable system modeling has been extensively studied since the 1960s [7-16], little research on RDT for repairable systems has been reported. Singh and Swaminathan [8] derived the exact confidence interval of system availability assuming the exponential distributions for time-between-failures and time-to-repair. Usher and Taylor [9] showed the design of availability test plans that meet the stated levels of producer and consumer risks. Methods for demonstrating failure intensity and MTBF also have been investigated. These methods are similar to the methods used for non-repairable systems. For instance, MIL-STD-781C, as pointed out by Ascher [10], is designed for reliability tests for repairable systems. It presents a test design method similar to the beta-binomial equation (*i.e.* Eqn. (1)). It assumes that the failure process of a repairable system is modeled by an HPP, where the failure intensity is usually considered to be constant over time. According to this military standard, for equipment failures following HPP with failure intensity ρ, the probability of observing r failures in an accumulated operating time T is:

$$P(r) = \left(\rho T\right)^r \left(e^{-(\rho T)} / r!\right)$$

Accordingly, the RDT design formula can be expressed as:

$$1 - CL = \sum_{i=1}^{r} (\rho T)^r \left(e^{-(\rho T)} / r!\right) \tag{4}$$

where CL is the confidence level. Similar to Eqn. (1), there are four parameters in the above equation: CL, ρ, T and r. Again, if any three out of the four variables are given, the remaining variable can be obtained. However, Blumenthal, et al. [11] pointed out that this method can lead to unacceptably high consumer risk when it is applied to a non-homogeneous process where some components in the system are aging.

In this paper, we propose a new method for non-repairable system test design that is based on the binomial equation. This method can be effectively applied to both HPP and NHPP cases.

2 METHODOLOGY OF RDT FOR REPAIRABLE SYSTEMS

In this paper, a repairable system is assumed to experience minimal repairs, which means the failure process follows NHPP. Many NHPP models have been developed, such as the log-linear failure intensity model by Cox and Lewis [7], the power law model by Crow [14], and the bounded failure intensity models by Pulcini [15] and Attardi and Pulcini [16]. In this paper, we use the power law model, which is the most popular NHPP model.

2.1 RDT Design Based on Number of Failures

Before we discuss the details of RDT for repairable systems, we will briefly introduce reliability growth analysis and repairable system analysis. As shown in Figure 1, a product usually needs to undergo several development stages before it is launched to market.

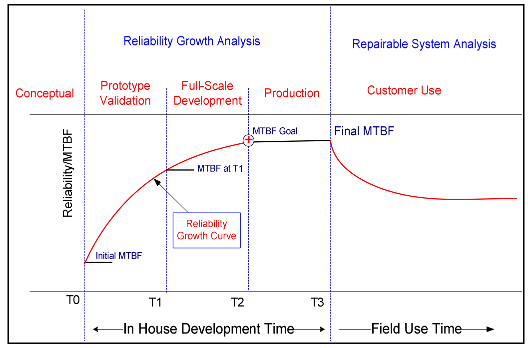

Figure 1- Product Development Stages

Among all of the stages shown in this figure, customers care only about the final achieved MTBF at the final development stage and the MTBF curve during field use. MIL-STD-781 and its newer version MIL-HDBK-781A [12] provide test methods and test plans that can be used in reliability testing during the development, qualification and production of repairable systems. Although the power law NHPP model is included in the standard for reliability growth analysis, the test methods included in MIL-HDBK-781A are based on the HPP assumption. This assumption sometimes may not be appropriate for operational tests on fielded units. For some systems, even when the achieved MTBF at the end of development (*i.e.* production stage in Figure 1) is high, customers will not be satisfied with the performance of the product if the MTBF decreases very quickly during field use. Therefore, operational tests and accelerated tests are usually conducted after the development stages to study the product's field reliability using one of previously mentioned NHPP models. For instance, the power law NHPP (also called Crow-AMSAA) model for the failure intensity $\rho_i(t)$ is:

$$\rho_i(t) = \theta\beta t^{\beta-1} \tag{5}$$

where θ and β are the model parameters. The instantaneous MTBF is defined as:

$$MTBF_i(t) = 1/\rho_i(t) \tag{6}$$

The average (also called cumulative) failure intensity and MTBF are:

$$\rho_c(t) = \int_0^t \rho_i(t)\, dt \Big/ t = \int_0^t \theta\beta t^{\beta-1} dt \Big/ t = \theta t^{\beta-1} \tag{7}$$

$$MTBF_c(t) = 1/\rho_c(t) = 1/\theta t^{1-\beta} \tag{8}$$

From the above equations, one can see that the failure intensity, average failure intensity and MTBF are calculated from the model parameters. This means that the requirements for these reliability metrics can be mathematically converted to the requirement for the model parameters. Therefore, the

RDT can be designed based on the requirement for the model parameters.

Similar to Eqn. (1), which is used for non-repairable systems, we propose the following equation for RDT for NHPP repairable systems:

$$1-CL=\sum_{i=0}^{r}\frac{(m\theta T^{\beta})^{i}\exp(-m\theta T^{\beta})}{i!} \quad (9)$$

where m is the number of systems to be tested, T is the required total test time for each system and r is the maximum number of failures allowed in the test. For an NHPP, the number of failures up to time T follows a Poisson distribution. Therefore, the expected cumulative number of failures resulting from all tested systems can be expressed as:

$$N(T)=m\int_0^T \rho_i(t)\,dt=m\theta T^{\beta} \quad (10)$$

If β is given, θ can be calculated based on a specific reliability criterion. β represents the failure behavior of a system and usually can be obtained from historical or test data. For example, if β is 2 and the criterion for the test is that the average MTBF by the end of 10 years of operation will be at least 0.5 years, then from Eqn. (8) we can solve θ by:

$$MTBF_c(t)=1/\theta\, t^{1-\beta} \Rightarrow 0.5=1/\theta\times 10^{-1} \Rightarrow \theta=0.2$$

By conducting the above procedure, we convert a specific reliability requirement to the requirement for model parameter θ. In Eqn. (9), there are four variables: T, m, r and CL. If any three out of the four are given, the remaining variable can be determined in order to demonstrate if θ can meet the requirement. If both r and β in Eqn. (9) are given, it can be proven that $2m\theta T^{\beta} \sim \chi^2_{2(r+1)}$. The proof is given in Appendix A. From the distribution of $2m\theta T^{\beta}$, we can get any percentile of parameter θ and also the percentile of its functions such as the MTBF or number of failures. Eqn. (9) can be rewritten as (see Appendix A):

$$2m\theta_{CL}T^{\beta}=\chi^2_{CL,2(r+1)} \quad (11)$$

For the above example, demonstrating that the average MTBF is at least 0.5 years is equivalent to demonstrating that θ is at most 0.2 with the same confidence level. Suppose that 6 systems are tested simultaneously (m = 6) and CL is 80% with the allowed number of failures of 2, we can solve the required test time for each system as:

$$2m\theta_{CL}T^{\beta}=\chi^2_{CL,2(r+1)} \Rightarrow T=1.888 \quad (12)$$

The result indicates that each system needs to be tested for 1.888 years in order to demonstrate the required MTBF. From Eqn. (12), we also know that 0.2 is the demonstrated 80% upper bound for θ when 6 of such a system is tested for 1.888 years. Similarly, its median (50% percentile) is:

$$2m\theta_{0.5}T^{\beta}=\chi^2_{0.5,2(r+1)} \Rightarrow \theta_{0.5}=0.12498 \quad (13)$$

2.2 Determination of Sample Size and Test Duration

In RDT, when the sample size is given, the test duration can be determined and vice versa. From Eqns. (9) and (11), one can see that sample size m and test duration T are linked by β, the required MTBF (or θ), confidence level and the number of failures allowed in the test r. Either of the two equations can be used to show the relationship between the sample size and the test duration. For example, Figure 2 provides the contour plot for a specific RDT design. The objective of this RDT is to demonstrate that the cumulative MTBF of the product in 10,000 hours of operation is 500 hours with a confidence level of 90%, where β = 1.2 is assumed and required θ is 0.000317, which can be obtained from Eqn. (8) using the requirement for MTBF.

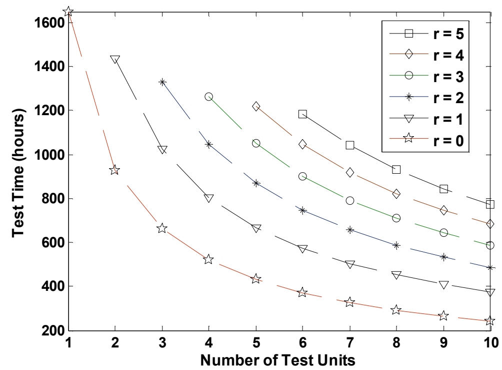

Figure 2- Contour Plots of Sample Size vs. Testing Time

Figure 2 shows the relation between the sample size, number of failures and testing time for RDTs that can be used to demonstrate the required MTBF for this example. Each curve in Figure 2 is for a fixed number of observed failures. For instance, with m = 4 available test units, if no failure is observed in the test (r = 0), the corresponding test duration is T = 520 hours.

2.3 RDT Design Based on Failure Time

Section 2.1 and 2.2 explained the proposed failure-number-based RDT for repairable systems and its application. In fact, the alternative method of failure-time-based RDT has been studied by some researchers [13]. When failure times are used to fit the power law NHPP model, the Maximum Likelihood Estimates (MLE) of the model parameters can be obtained as:

$$\hat{\theta}=\sum_{j=1}^{m}n_j\Big/\left(mT^{\hat{\beta}}\right);\quad \hat{\beta}=\sum_{j=1}^{m}n_j\Bigg/\left[\sum_{j=1}^{m}n_j\ln T-\sum_{j=1}^{m}\sum_{i=1}^{n_j}\ln t_{j,i}\right]$$

The approximate variance-covariance matrix of the estimated model parameters [$\hat{\beta},\hat{\theta}$] can be expressed as:

$$\Sigma=\begin{bmatrix}\mathrm{var}(\hat{\beta}) & \mathrm{cov}(\hat{\beta},\hat{\theta})\\ \mathrm{cov}(\hat{\beta},\hat{\theta}) & \mathrm{var}(\hat{\theta})\end{bmatrix}=I^{-1}$$
$$=\beta^2\Big/\left(mT^{\beta}\right)\begin{bmatrix}1/\theta & -\ln T\\ -\ln T & \theta/\beta^2+\theta\ln^2 T\end{bmatrix} \quad (14)$$

where I is the associated Fisher information matrix. On the other hand, if β is given as discussed earlier and only θ needs to be estimated, the MLE of θ and its variance can be obtained as:

$$\hat{\theta}=\sum_{j=1}^{m}n_j\Big/\left(mT^{\beta}\right);\ \text{and}\ \mathrm{var}(\hat{\theta})=\hat{\theta}\Big/\left(mT^{\beta}\right) \quad (15)$$

The total number of failures follows a Poisson distribution for an NHPP. Since a Poisson distribution can be approximated by a normal distribution, we assume that $\hat{\theta}$

follows a normal distribution with the mean and variance given in Eqn. (15). As a result, the upper bound of θ is:

$$\theta_{CL} = \hat{\theta} + z_{CL} std(\hat{\theta}) \quad (16)$$

where Z_{CL} is the CL percentile of the standard normal distribution. The accuracy of the approximation will be studied through simulation in the next section. To estimate the expected value of the confidence bound using Eqn. (16), $\hat{\theta}$ is replaced by a pivot value (planning value) of θ for the test design. Consider the same example shown in Section 2.1. If we use the median value calculated in Eqn. (13) for θ, the upper one-sided confidence bound for θ becomes:

$$\theta_{0.8} = \hat{\theta} + z_{CL} std(\hat{\theta}) \Rightarrow \theta_{0.8} = 0.1893 \quad (17)$$

Therefore, if the RDT design method based on failure time is used, the demonstrated 80% upper bound for θ is 0.1893, which is close to the value of 0.2 obtained in Section 2.1. Appendix B mathematically proves that the Fisher matrix bound of Eqn. (16) will indeed lead to results similar to the ones from the proposed RDT design method based on number of failures. Comparisons between these two design methods and the results obtained from simulation are given next.

3 CASE STUDY AND COMPARISON OF DIFFERENT METHODS

An engineer is required to design a test to demonstrate that the total number of failures for a system would be less than 20 during 10,000 hours of operation. In other words, the required average MTBF is longer than 500 hours. The required confidence level is 80% and the engineer has only 400 hours for the test. The failure process is assumed to follow a power law NHPP with β = 1.2. Given the available resources, no more than 6 failures are allowed during the test. The design problem is to determine the sample size to demonstrate that the system meets the requirement.

First, we will apply the proposed RDT method in this paper. For the power law NHPP, the expected number of failures is:

$$N(t) = \theta t^{\beta}$$

Since the required number of failures during 10,000 hours of operation must be less than 20, we can convert this requirement to the requirement for model parameter θ. So the required corresponding upper bound for θ is:

$$\theta_{CL} = N(t)_{CL} / t^{\beta} = 20/10000^{1.2} = 3.1698E-4 \quad (18)$$

From Eqn. (11), we have:

$$2m\theta_{CL} T^{\beta} = \chi^2_{CL,2(r+1)}$$

Therefore, the sample size m is calculated by:

$$m = \chi^2_{CL,2(r+1)} / \left(2T^{\beta}\theta_{CL}\right) = 21.595$$

which indicates that 22 units are needed for the test. From Eqn. (13), we can also calculate the median of θ as:

$$\theta_{0.5} = \chi^2_{0.5,2(r+1)} / (2mT^{\beta}) \Rightarrow \theta_{0.5} = 2.3295E-4 \quad (19)$$

Second, we compare the above results with the results from the existing RDT method based on failure time. If we test 22 units and use the median as the pivot value for θ in Eqn. (16), then the upper bound for θ can be obtained as:

$$\theta_{CL} = \hat{\theta} + z_{CL} std(\hat{\theta}) \Rightarrow$$
$$\theta_{CL} = \hat{\theta} + z_{CL}\sqrt{\hat{\theta}/\left(mT^{\beta}\right)} \Rightarrow \theta_{CL} = 3.0887E-4 \quad (20)$$

Using this value, we can get the expected lower bound for the average MTBF at 10,000 hours:

$$MTBF_{CL} = 1/\theta_{CL}\, t^{1-\beta} = 513.129 \quad (21)$$

This value is close to the required MTBF of 500 hours that was obtained using the proposed RDT. From the above case study, one can see that the RDT method based on number of failures proposed in Section 2.1 yields similar results to the existing failure-time-based method in Section 2.2. However, the proposed method is much simpler and easier to use. This is not just because only one equation, Eqn. (11), is needed, but also because its theory is similar to the well-known RDT method for non-repairable systems. This property makes it is easy to be understood and accepted by engineers.

Finally, let us also compare the results of the two design methods against the results obtained through simulation. For the case study, the simulation inputs are:

- Number of systems m = 22
- $\beta = 1.2$; $\theta = 2.3295E$-4 (the median of θ)
- Number of simulations = 10,000 with seed = 2

	Median of θ	80% upper bound of θ	80% lower bound of MTBF
Proposed Method Based on Number of Failures (Analytical)	2.3295E-4	3.1698E-4	500
Method Based on Failure Time (Analytical)	2.3295E-4	3.0887E-4	513.1292
Simulation	2.3999E-4	3.0857E-4	513.6321

Table 1- Analytical and Simulation Results

The standard deviation of the simulated θs in Table 1 is 8.8679E-5. To get the median and the 80% upper bound of θ, the 10,000 simulated values of θ are sorted first. Then, the value at the 5,000th position is identified as the median and the value at the 8,000th position is identified as the 80% upper bound of θ. Here the median is shown in Table 1 because we want to compare the simulated median with the true value of θ used in the simulation. The closer these two values, the more accurate the simulation is. θ and MTBF have a one-to-one relationship, the median and the lower bound of MTBF can be calculated based on the corresponding value of θ. Since the purpose of this RDT is to determine the sample size in order to demonstrate the 80% lower bound of the MTBF, the 80% bounds obtained from all the methods are displayed in Table 1. From the table, it can be seen that the proposed RDT method is valid since the number of test units obtained from the new method (m = 22) is confirmed by both the existing method and the simulation results.

4 CONCLUSIONS

In this paper, the similarity between the RDT design methods for non-repairable and repairable systems is discussed first. Test design methods based on number of failures and failure time are available for both types of systems. However, the existing method based on number of failures is suitable only for HPP repairable systems. This paper proposes a RDT design method that is applicable for both HPP and NHPP repairable systems. The accuracy of the proposed method is compared against the existing traditional method and is also validated through simulation. The existing method based on failure time needs to estimate the model parameters and uses the Fisher information matrix to calculate the parameter bounds. Although it can be used for NHPP, it has several drawbacks. For instance, Fisher bounds are only good for large sample sizes; hence, the normal distribution assumption for θ may not be accurate in some situations. The proposed method does not have the above issues. Moreover, both the analytical analysis and simulation results show that the proposed method is efficient and easy to use.

5 APPENDICES

5.1 Appendix A

For Eqn. (9):

$$1-CL=\sum_{i=0}^{r}\frac{(m\theta T^{\beta})^{i}\exp(-m\theta T^{\beta})}{i!} \tag{A.1}$$

Let $x=m\theta T^{\beta}$, then the above equation becomes:

$$1-CL=\sum_{i=0}^{r}\frac{x^{i}e^{-x}}{i!} \tag{A.2}$$

The lower confidence bound for cumulative MTBF is the higher confidence bound for x. Therefore, the above equation can be written as:

$$\Pr(X<x)=CL=1-\sum_{i=0}^{r}\frac{x^{i}e^{-x}}{i!} \tag{A.3}$$

This is the cumulative distribution function (*cdf*) of *x*.

For a Gamma distribution $y\sim Gamma(k,\lambda)$, its *cdf* is:

$$F(y;k,\lambda)=1-\sum_{i=0}^{k-1}\frac{(\lambda y)^{i}e^{-\lambda y}}{i!} \tag{A.4}$$

Comparing Eqn. (A.3) with Eqn. (A.4), one can see that x follows the Gamma distribution $x\sim Gamma(r+1,1)$ and $2x\sim Gamma(r+1,2)$. It is known that $\chi^2_{2(r+1)}$ is the same as a Gamma distribution $Gamma(r+1,2)$. Therefore, we can also say that $2x\sim\chi^2_{2(r+1)}$. Since $x=m\theta T^{\beta}$, we have:

$$2m\theta T^{\beta}\sim\chi^2_{2(r+1)}$$

5.2 Appendix B

From the RDT method based on number of failures and Eqn. (13), we known the median of θ is:

$$\theta_{0.5}=\chi^2_{0.5,2(r+1)}\Big/\left(2mT^{\beta}\right) \tag{B.1}$$

and its upper bound at confidence level of *CL* is:

$$\theta_{CL}=\chi^2_{CL,2(r+1)}\Big/\left(2mT^{\beta}\right) \tag{B.2}$$

From the methods based on failure time and the Fisher bound of Eqn. (16), we know the upper bound of θ is:

$$\hat{\theta}+z_{CL}std(\hat{\theta}) \tag{B.3}$$

Using the median of θ in Eqn. (B.1) as the pivot value in Eqn. (B.3), Eqn. (B.3) can be rewritten as:

$$\frac{\chi^2_{0.5,2(r+1)}}{2mT^{\beta}}+z_{CL}\frac{\sqrt{2\chi^2_{0.5,2(r+1)}}}{2mT^{\beta}} \tag{B.4}$$

We need to prove that Eqn. (B.2) and Eqn. (B.4) will result in similar values. In other words, we need to prove that:

$$\Delta=\left(\frac{\chi^2_{0.5,2(r+1)}}{2mT^{\beta}}+z_{CL}\frac{\sqrt{2\chi^2_{0.5,2(r+1)}}}{2mT^{\beta}}\right)\Bigg/\left(\frac{\chi^2_{CL,2(r+1)}}{2mT^{\beta}}\right)\approx 1$$

Since mT^{β} is constant, the equation is equivalent to:

$$\Delta=\left(\chi^2_{0.5,2(r+1)}+z_{CL}\sqrt{2\chi^2_{0.5,2(r+1)}}\right)\Big/\left(\chi^2_{CL,2(r+1)}\right)\approx 1 \tag{B.5}$$

For a Chi-Squared distribution, its percentile can be approximated by:

$$\chi^2_{\alpha,v}=v\left(1-\frac{2}{9v}+z_{\alpha}\sqrt{\frac{2}{9v}}\right)^{3}$$

where α is the left tail percentile and v is the degree of freedom. Applying this approximation in Eqn. (B.5) and letting $2(r+1)=v$, we can rewrite the equation as:

$$\Delta=\frac{v(1-\frac{2}{9v})^{3}+z_{CL}\sqrt{2v(1-\frac{2}{9v})^{3}}}{v\left(1-\frac{2}{9v}+z_{CL}\sqrt{\frac{2}{9v}}\right)^{3}} \tag{B.6}$$

Since $2/9v$ is a value close to 0, it can be ignored in the calculation. For instance, if there are 2 failures, its value is only 0.037. So, the numerator of Eqn. (B6) becomes:

$$v(1-\frac{2}{9v})^{3}+z_{CL}\sqrt{2v(1-\frac{2}{9v})^{3}}\approx v+z_{CL}\sqrt{2v} \tag{B.7}$$

Similarly, the denominator becomes:

$$\begin{aligned}&v\left(1-\frac{2}{9v}+z_{CL}\sqrt{\frac{2}{9v}}\right)^{3}=v\left(1+z_{CL}\sqrt{\frac{2}{9v}}\right)^{3}\\&\approx v\left(1+3z_{CL}\sqrt{\frac{2}{9v}}\right)=v\left(1+z_{CL}\sqrt{\frac{2}{v}}\right)=v+z_{CL}\sqrt{2v}\end{aligned} \tag{B.8}$$

where $(1+x)^{n}\approx 1+nx$ when x is close to 0. Therefore, the ratio of the upper bounds from the two different test design methods is:

$$\Delta=\frac{v(1-\frac{2}{9v})^{3}+z_{CL}\sqrt{2v(1-\frac{2}{9v})}}{v\left(1-\frac{2}{9v}+z_{CL}\sqrt{\frac{2}{9v}}\right)^{3}}\approx\frac{v+z_{CL}\sqrt{2v}}{v+z_{CL}\sqrt{2v}}=1 \tag{B.9}$$

This shows that the two methods will yield similar results although they are based on different theories.

REFERENCES

1. Kececioglu, D., *Reliability & Life Testing Handbook*, vol. 2. Prentice-Hall, Inc., 1994.
2. Leemis, L. M., "Lower System Reliability Bounds From Binary Failure Data Using Bootstrapping." *Journal of Quality Technology*, (2006): 38(1), 2-13.
3. Guo, H., Honecker, S., Mettas, A., and Ogden, D., "Reliability Estimation for One-Shot Systems with Zero

Component Test Failures, *2010 Annual Reliability and Maintainability Symposium*.
4. Lawless, J. F., "Confidence Interval Estimation for the Weibull and Extreme Value Distributions," *Technometrics* (1978): 20, 355-364.
5. McKane, S. W., Escobar, L. A. and Meeker, W. Q., "Sample Size and Number of Failure Requirements for Demonstration Tests with Log-Location-Scale Distributions and Failure Censoring," Technometrics (2005): 47, 182-190.
6. Nelson, W. and Meeker, W., "Theory for Optimum Accelerated Censored Life Tests for Weibull and Extreme Value Distributions," *Technometrics* (1978): 20, 171-177.
7. Cox, D. R. and Lewis, P. A. W., *The Statistical Analysis of Series of Events*. London: Methuen, (1966)
8. Singh, V. P. and Swaminathan, S., "Sample Size for System Availability," *2002 Annual Reliability and Maintainability Symposium*, 2002.
9. Usher, J. and Taylor, G., "Availability Demonstration Testing," *Quality and Reliability Engineering International* (2006): 22, 473-479.
10. Ascher, H., "MIL-STD-781C: A Vicious Circle," *IEEE Transactions on Reliability* (1987): R-36, 397-402.
11. Blumenthal, S., Greenwood, J., and Herbach, L., "Series Systems and Reliability Demonstration Tests," *Operations Research* (1984): 32(3), 641-648.
12. MIL-HDBK-781A (1996), "Reliability Testing for Engineering Development, Qualification, and Production." Department of Defense, Washington, D. C., 1996.
13. Guo, H. and Pan, R., "On Determining Sample Size and Testing Duration of Repairable System Life Testing," *2008 Annual Reliability and Maintainability Symposium*.
14. Crow, L. H., *Reliability Analysis for Complex Repairable Systems, in Reliability and Biometry* ed. by F. Proschan and R.J. Serfling, 379-410. SIAM, 1974.
15. Pulcini, G., "A Bounded Intensity Process for the Reliability of Repairable Equipment." *Journal of Quality Technology* (2001): 33, 480-492.
16. Attardi, L. and Pulcini, G., "A New Model for Repairable Systems with Bounded Failure Intensity," *IEEE Transactions on Reliability* (2005): 54(4), 572-582.

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