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Improved Reliability Using Accelerated Degradation & Design of Experiments

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Key Words: Accelerated Degradation, Design of Experiments, Linear Regression, Log-normal Distribution

SUMMARY & CONCLUSIONS

For many products, an underlying degradation process is the root cause of failures. Often, such degradation processes are not directly observable and all that can be learned from the test is the time of failure. In order to get useful information in a short time, accelerated degradation testing is frequently used. Most of the existing degradation analysis methods assume that the degradation process can be regularly inspected and the degradation amount can be easily and accurately measured. Unfortunately, for many products and many testing processes, this is not an easy task.

In this paper, a design of experiment (DOE) method of using the degradation process together with the observed failure data to improve reliability is proposed. Unlike other degradation analysis methods, the proposed method does not require regular degradation measurements. In the use of DOE, all the factors that affect the degradation process are classified into two types. The Type I factor is called the amplification factor. Its effect on degradations is well known based on the engineering knowledge of the physical process of the degradation. This factor is used to amplify (accelerate) the degradation process. The Type II factors are called control factors. Their effects are unknown and need to be studied by experiments. By combing the engineering knowledge and the observed failures, the effects of control factors are analyzed using a linear regression method. Important effects and the optimum settings of control factors are identified. The product reliability can be improved by operating under the optimum settings.

1 INTRODUCTION

Many degradation modeling methods have been proposed in recent years. Lu and Meeker [1] developed statistical methods of using degradation measures to estimate a time-to-failure distribution for a broad class of degradation models. Lu, Park and Yang [2] proposed a model with random regression coefficients and a standard deviation function for analyzing linear degradation data. In order to reduce the experiment time and cost, Meeker, Escobar and Lu [3] accelerated the degradation process by applying high temperatures on the test units. The acceleration degradation process then is used to establish the relationship between failure times and degradation values. Simulation-based methods are used to compute confidence intervals for quantities of interest (e.g., failure probabilities). Other degradation models and failure time prediction methods also can be found. For example, Essayed and Liao [4] applied a Geometric Brownian Motion model for field degradation data; Bae and Kvam [5] used a nonlinear random-coefficients model for degradation testing.

In additional to finding a better modeling method for degradation processes, some initial research works of using degradation data to improve product reliability were also conducted. Chiao and Hamada [6] modeled the degradation process as a linear function of time under different control factor settings. Failure time is defined as the point when the degradation measurement is higher than a pre-determined threshold. Based on the path of the degradation process and the threshold for defining a failure, the failure time of each test unit can be predicted. Then the relationship between failure times and control factors can be identified and therefore, the reliability can be improved by applying suitable factor levels for the products. Similar work was done by Tseng, Hamada and Chiao [7]. Instead of modeling the degradation process only as a function of time and then establishing the relationship between experiment factors and the predicted failure times, Scibilia et al. [8] suggested using the slope of the degradation function as a response. After building the relationship between the degradation rate and experiment factors, the degradation rate can be reduced by operating the product under certain factor levels, and the reliability of the products can be improved.

Clearly, using degradation data to improve product reliability has great potential because the existing quality techniques such as regression, time series modeling, and DOE can be integrated with reliability analysis methods to finally improve the product quality and reliability. In this paper, a DOE method of using the degradation process together with the observed failure data is developed to improve the product reliability. The advantage of the proposed method is that it can be used in the case where the degradation cannot be easily measured, especially under accelerated testing circumstances.

This paper is organized as follows: In Section 2 the motivation behind and the background of the degradation amplification method are discussed. Models for connecting the degradation with the failure probability are developed. Section 3 presents the case studies. Further discussion and conclusions are given in Section 4.

2 MODELS FOR DEGRADATION FAILURE DATA

It is very popular to use accelerated testing to study product reliability. Units are tested at high levels of one or several stresses to “accelerate” one or more well-understood failure mechanisms (e.g., wear, cracking, chemical change, and diffusion). Possible accelerating stresses include increased use rate, temperature, humidity, pressure, voltage,

0-7803-9766-5/07/$25.00 ©2007 IEEE

and ultraviolet (UV) light intensity. For many products, the relationship between the acceleration stress and the degradation can be easily established based on engineering knowledge. However, besides the acceleration stress, other factors may also affect the degradation process, and the effects of these factors are not clear. In order to estimate their effects, experiments should be carefully designed and conducted. For example, suppose that it is known that the temperature is the most important factor that will affect the degradation process for a given product. But it is not clear whether other factors, such as the two choices of material and the two choices of fabrication methods, will affect the degradation process or not. Therefore, a 2 by 2 full factorial design on the material and the fabrication method can be conducted. In this experiment, temperature can be used as the amplification or acceleration factor to amplify the degradation process.

Let's define the *acceleration factor* as *Type I factor*. Its effect on the degradation is well known based on engineering knowledge. Therefore this factor is not directly of interest to the experimenters. The other related factors whose effects are under investigation are called *Type II factors* or *control factors*. Failures related to degradation are also classified into two classes. If there is a threshold and when the degradation amount is above the threshold, observable failures occur, this type of failure is called *hard failure*. If there is no clear threshold and the probability of failure is proportional to the degradation amount, this failure is called *soft failure*. In this paper, we will only focus on the hard failures. Modeling method for soft failures will be studied in the future research.

Let Y be the degradation measurement, X be the control factors and M be the acceleration (amplification) factor. The degradation function is assumed as:

$$\ln(Y_t)=\gamma+\beta(X)Mt+\sigma(X)\varepsilon_t \tag{2.1}$$

where Y_t is the degradation value or the transformation of the degradation at time t. It should be noticed that in the testing condition, some factors in X also can be "noise factors". From this equation, it can be seen that the control factors X will affect the degradation rate through the amplification factors. Eq. 2.1 is used for illustration purposes. For different processes, based on the engineering knowledge, different degradation models that have more physical meaning should be used. The value of M also can be any transformation of the amplification factor, such as power, logarithm, or exponential transformation. Eq. 2.1 can be expanded as:

$$\begin{aligned}\ln(Y_t)&=\gamma+\left(\beta_0+\sum\beta_i x_i+\sum\sum\beta_{il}x_i x_l\right)Mt\\&\quad+\left(\lambda_0+\sum\lambda_i x_i+\sum\sum\lambda_{il}x_i x_l\right)\varepsilon_t\\&=\gamma+\mu(X,M)t+\sigma(X)\varepsilon_t\end{aligned} \tag{2.2}$$

If Y_t is log-normally distributed, then the ε_t are i.i.d. (independent and identically distributed) following the normal distribution with mean value of 0 and standard deviation of 1. The location-scale-distribution model in Eq. 2.2 assumes that factors (amplification factor and control factors) and time affect the mean value of $\ln(Y_t)$. Only the control factors affect the scale parameters through $\sigma(X)$. So the variance is a constant value if the setting of X is the same. However, in many applications, the error term in Eq. 2.2 is usually assumed to be a constant value under all the settings of X and is independent of time, for example, in the models used by Tseng, Hamada and Chiao [7] and Meeker, Escobar and Lu [3]. In this paper, constant variance will be used and Eq. 2.2 then reduces to:

$$\ln(Y_t)=\gamma+\mu(X,M)t+\sigma\varepsilon_t \tag{2.3}$$

According to the definition of hard failures, the probability of failure at time t is:

$$\Pr(failure)=\Pr(Y_t>S) \tag{2.4}$$

where S is the threshold value. Observable failure occurs without measuring the degradation amount if the degradation is greater than S. Assume the degradation at time t is log-normally distributed, then:

$$\Pr(Y_t>S)=\Phi\left(\frac{\gamma+\mu(X,M)t-\ln S}{\sigma}\right) \tag{2.5}$$

Since the degradation is not measurable or is difficult to be measured during experiments, the only available information is the number of failures at each inspection time. Based on the failure information, the probability of failure at time t can be estimated using the Median Rank method or Kaplan-Meier method [9]. Using a two level full factorial design, the failure data type can be illustrated as:

Run	Amp. Factor	Control factors			Inspection Time (record # of failures)		
	M	A	B	AB	T1	T2	Tk
1	m_1	-	-	+	r_{1T_1}	r_{1T_2}	r_{1T_k}
2	m_2	+	-	-	r_{2T_1}	r_{2T_2}	r_{2T_k}
3	m_3	-	+	-	r_{3T_1}	r_{3T_2}	r_{3T_k}
4	m_4	+	+	+	r_{4T_1}	r_{4T_2}	r_{4T_k}

Table 2.1 - Failure Data for Hard Failures

In Table 2.1, r_{jT_i} is the number of failures at inspection time T_i under the jth run. If the total number of units of the jth run is n_j, the probability of failure at time T_i is:

$$p_{jT_i}=\frac{r_{jT_i}-0.3}{n_j+0.4} \tag{2.6}$$

which is the approximated median rank result. Eq. 2.5 is the predicted probability of failures based on the degradation functions, while Eq. 2.6 is the observed value. Therefore, these equations can be connected together to estimate the model parameters. Set Eq. 2.6 equal to Eq. 2.5, we have

$$p_{jT_i}=1-\Phi\left(\frac{\ln(S)-\gamma-\mu(X_j,m_j)T_i}{\sigma}\right)$$

$$\Rightarrow\Phi^{-1}\left(1-p_{jT_i}\right)=\frac{\ln(S)-\gamma-\mu(X_j,m_j)T_i}{\sigma}$$

$$\Rightarrow\Phi^{-1}\left(1-p_{jT_i}\right)=a+\alpha(X_j,m_j)T_i+b\ln(S) \tag{2.7}$$

where:

$$a=-\frac{\gamma}{\sigma};\qquad \alpha(X_j,m_j)=-\frac{\mu(X_j,m_j)}{\sigma};\qquad b=\frac{1}{\sigma}$$

$$\mu\left(X_j, m_j\right) = \left(\beta_0 + \sum \beta_i x_{j,i} + \sum\sum \beta_{il} x_{j,i} x_{j,l}\right) m_j$$

and $x_{j,i}$ is the value of factor i under the j th run.

Eq. 2.7 is a group of linear equations. So the model parameters can be estimated using the Least Squares Estimation (LSE) method. It also can be estimated using Maximum Likelihood Estimation (MLE) method. The effect of each control factor can be studied using the regular ANOVA method or Likelihood Ratio Test method. Be aware that, this proposed method can be easily extended to more complicated cases where more factors are involved and more complicated degradation functions are used.

3 CASE STUDY

An example of a two-level full factorial design will be discussed in this section. This example illustrates how the methods proposed in Section 2 can be easily used to improve product reliability, compare different designs and even optimize the product design.

A design group is developing a new product. There are two choices of the product materials and two choices of the fabrication methods. They know that the life of the product is related with a degradation process. If the degradation amount is above a given threshold, observable failures will occur. In order to make the right choice to ensure the product reliability, the design group asked the reliability group to help them to identify the effect of material, fabrication method and their interaction effect on the product reliability. It is known that voltage level can affect the degradation rate. The reliability group decided to apply an accelerated degradation test to find the answer in a short time. The experiment is designed as:

- Amplification factor: Voltage, continuous variable;
- Control factors: Material (A = 1, Material 1; A = -1, Material 2), Fabrication Method (B = 1, Method 1; B = -1, Method 2), 2-level discrete variable;
- Sample size: 150 at each run;
- Degradation threshold: ln(S) = 5.5;
- Inspection interval: every 5 hours, result in 5 observations.

The experiment results are given in Table 3.1

Run	Amp. Factor	Control Factors			Num of Failure at Inspection Time				
	M	A	B	AB	5	10	15	20	25
1	4	-1	-1	1	16	31	39	45	54
2	3.5	1	-1	-1	27	47	76	100	122
3	3	-1	1	-1	18	28	40	60	79
4	2.5	1	1	1	21	56	81	115	139

Table 3.1 - Two-factorial design with amplification factor

Applying the median rank method, the probability of failure at each inspection time is given in Table 3.2

The degradation amount at time t is assumed to follow the log-normal distribution; then Eq. 2.7 is applied.

$$\begin{aligned}\Phi^{-1}\left(1-p_{jT_i}\right) &= a + \left(\alpha_0 + \alpha_1 x_{iA} + \alpha_2 x_{iB} + \alpha_{1,2} x_{i,AB}\right) m_i t_j + b \ln\left(S\right) \\ &= a' + \left(\alpha_0 + \alpha_1 x_{iA} + \alpha_2 x_{iB} + \alpha_{1,2} x_{i,AB}\right) m_i t_j \qquad (3.1)\end{aligned}$$

where $a' = a + b\ln(S)$. The regression ANOVA table is given in Table 3.3.

	Run	Probability of Failure at each Inspection Time				
		5	10	15	20	25
p_{jT_i}	1	0.1044	0.2041	0.2573	0.2972	0.3570
	2	0.1775	0.3105	0.5033	0.6629	0.8092
	3	0.1177	0.1842	0.2640	0.3969	0.5233
	4	0.1376	0.3703	0.5366	0.7626	0.9222

Table 3.2 - Probability of failure for hard failure example

Source	DF	SS	MS	F	P
Regression	4	9.8326	2.4581	311.32	0
Residual Error	15	0.1184	0.0079		
Total	19	9.951			

Table 3.3 - Regression ANOVA for the Hard Failure Example

The adjusted R-square value of this regression result is 98.5%. Therefore, this linear regression equation can explain the experiment results very well. The significance of each regression term can be further examined. The coefficients of the linear equation are given in Table 3.4.

Term	Coef	Std of Coef	T	P
a'	1.47724	0.0466	31.7	0
a_0	-0.02602	0.000894	-29.1	0
α_1 (A)	-0.00994	0.000392	-25.4	0
α_2 (B)	-0.00628	0.000406	-15.48	0
$\alpha_{1,2}$ (AB)	-0.00233	0.000387	-6.02	0

Table 3.4 - Regression Coefficients for the Hard Failure Example

From Table 3.4, it can be seen that both the effect of material and the fabrication method are significant. Therefore, the significance of the interaction term is caused by the materials. From Eqs. 3.1 and 2.7, we know the expected degradation rate coefficient under j^{th} each setting is:

$$d_j = -\left(\alpha_0 + \alpha_1 x_{jA} + \alpha_2 x_{jB} + \alpha_{1,2} x_{j,AB}\right)\sigma \qquad (3.2)$$

If at the usage stress, the value of voltage M is 1, the degradation rates under different settings are given in Table 3.5.

Setting	Usage Stress	Control Factors			Expected Degradation Rate (σ)
	M	A	B	AB	
1	1	-1	-1	1	0.012119
2	1	1	-1	-1	0.020032
3	1	-1	1	-1	0.027351
4	1	1	1	1	0.044568

Table 3.5 – Expected Degradation Rate at Each Setting

From Table 3.5, we can conclude that, the setting of A = -1 and B = -1, which is material type 2 and fabrication method 2, should be used. Under this setting, the smallest degradation rate can be obtained. In the use of Least Squares Estimation method, the model residuals should be i.i.d. normally distributed. The normal probability plot is given in Figure 3.1.

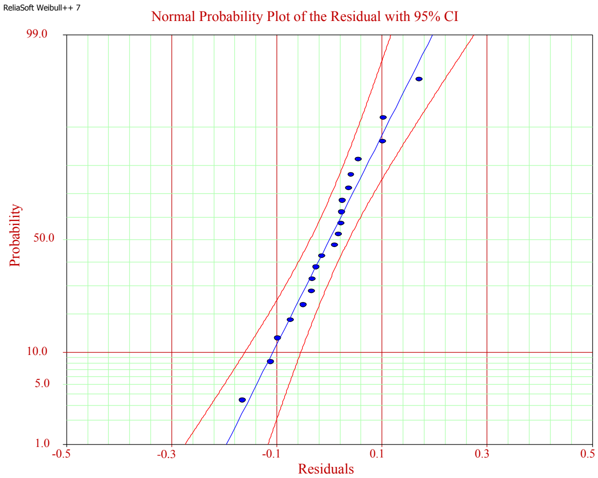

Figure 3.1 - Normal Probability Plot of the Residuals

From Figure 3.1, we cannot reject that the residuals follow the normal distribution with mean 0 and standard error is 0.084.

MLE also can be used to estimate the model parameters. In order to apply MLE, we rewrite the Eq. 3.1 as:

$$z_{jT_i} = \ln\left\{-\ln\left(p_{jT_i}\right)\right\} = \mu_{jT_i} + \sigma'\varepsilon_{jT_i} \qquad (3.3)$$

The likelihood function is:

$$L \propto \prod_j \prod_i \frac{1}{\sigma'} \exp\left[-\frac{1}{2}\left(\left(z_{jT_i} - \mu_{jT_i}\right)/\sigma'\right)^2\right] \qquad (3.4)$$

and the log likelihood function is:

$$\ln\left(L\right) \propto -N\ln\sigma' - \frac{1}{2}\left(\left(z_{jT_i} - \mu_{jT_i}\right)/\sigma'\right)^2 \qquad (3.5)$$

If MLE is used to estimate the model parameters, the likelihood ratio test can be used to test the significance of each effect. The LR statistic given by:

$$LR = -2\ln\left[\frac{\sup\left\{L_{0_k} : \underline{\theta}_k = \underline{\theta}_{0k}\right\}}{\sup\left\{L_{\theta_k} : \underline{\theta}_k \neq \underline{\theta}_{0k}\right\}}\right] \qquad (3.6)$$

which follows the chi-square $\left(\chi_k^2\right)$ distribution with *k* degrees of freedom if the hypothesis that $\underline{\theta}_k = \underline{\theta}_{0k}$ is true. $\sup\left\{L_{0_k} : \underline{\theta}_k = \underline{\theta}_{0k}\right\}$ and $\sup\left\{L_{\theta_k} : \underline{\theta}_k \neq \underline{\theta}_{0k}\right\}$ are the maximum likelihood values obtained for $\underline{\theta}_k = \underline{\theta}_{0k}$ and $\underline{\theta}_k \neq \underline{\theta}_{0k}$, respectively. $\underline{\theta}$ is the model parameter vector. It follows that the null hypothesis is rejected if $LR > \chi_{\alpha,k}^2$, in which $\chi_{\alpha,k}^2$ is the upper α quantile of the χ^2 distribution with *k* degrees of freedom. For example, if we want to test the significance of α_0. The value of *LR* can be calculated from the likelihood value of the unconstrained model and the model under the constraint $\alpha_0 = 0$. The MLE results are:

Term	MLE Coeff.	LR	P
a'	1.477252	84.39	0
α_0	-0.026018	81.02	0
α_1 (A)	-0.009942	75.68	0
α_2 (B)	-0.006282	56.62	0
$\alpha_{1,2}$ (AB)	-0.002326	24.56	0

Table 3.6 - MLE Coefficients for the Hard Failure Example

From Table 3.6, we see that the regression result and statistical test result are consistent with the Least Squares method.

4 CONCLUSIONS AND FUTURE WORK

In this paper, a general method to analyze degradation related failures was proposed. This method is developed to particularly handle the failures caused by degradation. By applying amplification factor together with control factors in the experiments, more information can be extracted and the effects of the control factors can be studied using the regular regression method. The proposed method can be applied in any design, such as fractional factorial design, Plackett – Burman design, and response surface method. If noise effect is considered in the degradation process, Taguchi method and cross design also can be used.

Different from the existing reliability studies using degradation data, the method in this paper does not require the regular measurement of the degradation. However, prior knowledge about the degradation path, which usually can be obtained through the physical explanation of the degradation process, is required.

In this paper, a linear function was used to describe the degradation process and only the log-normal distribution for hard failures is studied. Further research, including using other stochastic processes, such as Brownian process or Gaussian process to describe degradation processes can be conducted. Instead of using the log-normal distribution, other distributions, such as the Weibull, Gumbel, and Birnbaum-Saunders distributions also can be used. The model for soft failures will be studied in the future research.

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