Practical Methods for Modeling Repairable Systems with Time Trends and Repair Effects

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Key Words: Proportional Intensity Model, Crow-AMSAA Model, Nonhomogeneous Poisson Process, Minimal Repair, General Repair.

SUMMARY & CONCLUSIONS

Modeling and analysis of repairable systems have drawn a lot of attention in reliability and maintenance area. Earlier studies and results in this field usually as sume that a system after each repair is as same as new (perfect repair/maintenance) or as same as old (minimal repair/maintenance). These two assumptions are often found very limited uses in practical applications, most repair activities may realistically result in a complicated intermediate one (general repair or imperfect repair/maintenance).

Recently, statisticians and reliability engineers start to focus more on this type of repairable systems where repair actions do not bring the system to an as same as new condition but rather bring the state of a failed system to a level that is somewhere between new and the status prior to failure, and propose various models. However, as Guo, Ascher and Love (Ref.1) noticed, too much attention is paid to the invention of new models, with little thought; it seems, as to their applicability. Too little attention is paid to data collection and considering the usefulness of models for solving real reliability problems. To our best knowledge, these models are difficult to be used to solve engineering problems either because of the strong assumptions or the model complexity.

In this paper, we propose a practical model which is based on Proportional Intensity (PI) Model and virtual time concept, and explore the tools to analyze general repairable systems. The proposed model can incorporate the time trends, proportional failure intensity as well as the cumulative repair effects. More important, unlike the current models, the closed forms for all the reliability metrics can easily be obtained by solving differential equations, thus the reliability engineers can develop the Fisher Information Matrix or Likelihood Ratio confidence bounds based on the closed form expressions. The practical use of the proposed method is demonstrated by two real case studies. The results show that our proposed method is a very promising, efficient and practical approach with the potential of becoming very useful in industry and of leading to further generalization of repairable systems analysis.

1. *INTRODUCTION*

Most models for repairable systems have the forms of non-homogenous Poisson processes (NHPP), like the famous Crow-AMSAA model (Ref. 2) which uses power law function, and bounded intensity process (BIP) model (Ref. 3) which uses bounded exponential function. The functional form of Crow-AMSAA model is

$$\lambda(t) = \lambda\beta t^{\beta-1},\ \lambda,\beta > 0;\ t \ge 0 \tag{1}$$

Depending on the value of the shape parameter β , it can fit a wide range of failure processes. When $\beta < 1$, the failure intensity in Eq.(1) decreases with time and hence the model describes a reliability improvement situation. When $\beta > 1$, the failure intensity increases with time and so the model can be used to the situations in which a reliability deterioration is observed. Finally, when $\beta = 1$, the non-homogeneous Poisson process reduces to a homogeneous Poisson process with constant failure intensity.

Pulcini (Ref. 3) noticed that, the repeated application of the repair actions sometimes produces a finite bound for the increasing failure intensity for some repairable systems. Therefore, a bounded intensity function should be applied for this type of systems. The functional form of bounded intensity model is

$$\lambda(t) = \alpha(1 - e^{-\frac{t}{\beta}}),\ \alpha,\beta > 0;\ t \ge 0 \tag{2}$$

Unlike Crow-AMSAA, bounded intensity model can only used to model repairable equipments with reliability deterioration with time. When time goes to infinity, the failure intensity will be close to α , which is the upper bound of failure intensity

Both Crow-AMSAA and BIP models assume that repairs will not affect the system failure processes. After each repair, the system is only restored to the same state as it was just before the failure. This type of repair usually is called minimal repair. However, in reality, the system may be restored to a state that is better than the state just before the failure. Therefore, models which can consider the repair effects were proposed during recent years (Ref. 4-8). The most popular one is the proportional intensity model. It was proposed by Cox (Ref. 9) and used by Lawless and Thiagarajah (Ref. 4). The intensity function of proportional

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intensity model is

$$\lambda(t) = e^{\alpha+\beta t+\gamma(t-t_{N(t-)})} \quad (3)$$

where, $t_{N(t-)}$ is the most recent failure time. This model assumes that repairs do not change the baseline form of intensity function but shift the intensity "block" vertically along the intensity axis after each repair. In Eq. (3), the baseline is a log-linear function $e^{\alpha+\beta t}$.

Although the proportional intensity model can consider the repair effect, it is difficult to use in solving engineering problems. In Eq. (3), $t_{N(t-)}$ is unknown for a given future time t. Therefore, the model cannot be used to conduct prediction unless $t_{N(t-)}$ can be obtained by some ways, for example, by simulation. Furthermore, for this model, no closed forms exist for the future failure intensity, failure number and MTBF (mean time between failures) because of $t_{N(t-)}$. In contrast, Crow-AMSAA and bounded intensity model has closed forms, but does not consider the repair effect. In order to overcome the drawbacks of the current models, new models which can consider the repair effect and also can easily be used in prediction should be developed. In this paper, a practical method which combines the advantages of Crow-AMSAA model and proportional intensity model is proposed.

The rest of the paper is organized as follows. In Section 2 the proposed model is given. The method for parameter estimation will be presented in Section 3. Section 4 illustrates how to use the proposed model to predict future reliability metrics such as failure number and failure intensity for a given time. The confidence intervals of the prediction are also discussed in this section. Numerical examples and comparisons with traditional models are provided in Section 5. Section 6 concludes the work in this paper and discusses some extension for future research.

2. *THE GENERALIZED PROPORTIONAL INTENSITY MODEL*

The empirical form of the proposed model is

$$\lambda(t) = \lambda\beta t^{\beta-1} e^{\gamma N(t)} \quad (4)$$

where $N(t)$ is the failure number by time t. The physical meaning of the model is that after each repair, the failure intensity will change because of the repair. The cumulative repair effect is reflected by the term of $\gamma N(t)$. Obviously, if $\gamma>0$, the repair will have negative effects, the failure intensity will increase. If $\gamma<0$, the repair will have positive effect, repairs make system become better. When $\gamma=0$, there is no effect of the repair and the model reduces to the Crow-AMSAA model. Obviously, if Eq.(4) is used directly, it will have the same problems as the traditional proportional intensity model because of the discrete term $\gamma N(t)$. In order to avoid the problem, Eq.(4) is modified as:

$$\lambda(t) = \lambda\beta t^{\beta-1} e^{\gamma m(t)} \quad (4)$$

where $m(t) = E[N(t)]$ and has the form of

$$m(t) = \int_0^t \lambda(t)dt \quad (5)$$

In order to study the change of $\lambda(t)$ over time, the first derivative of $\lambda(t)$ with respect to time t is taken as:

$$\frac{d\lambda(t)}{dt} = \lambda\beta t^{\beta-2} e^{\gamma m(t)} \left[(\beta-1)+\gamma\lambda(t)t\right] \quad (6)$$

The sign of the derivative in Eq. (6) is decided by

$$(\beta-1)+\gamma\lambda(t)t \quad (7)$$

since all other terms are positive. Obviously, Eq. (7) can equal to, less than and greater than 0 at different time period. The sign of Eq. (7) is decided by the shape parameter of the baseline failure intensity, which is β, repair effect, which is γ and the current system state which is $\lambda(t)t$. If Eq. (7) is a positive value at time t, $\lambda(t)$ will be an increasing function. The system is deteriorating. If Eq. (7) is a negative value, $\lambda(t)$ will be a decreasing function. The system is improving. Finally, if Eq. (7) is close to 0 at some time periods, the system has constant failure intensity. From the above analysis, the flexibility of the proposed model is obvious.

From Eq. (4), the closed form of $m(t)$ and $\lambda(t)$ can be obtained. Rewrite Eq. (4) as:

$$\frac{dm(t)}{dt} = \lambda\beta t^{\beta-1} e^{\gamma m(t)} \quad (8)$$

and solve the above differential equation by the following procedure

$$\begin{aligned}
&\frac{1}{e^{\gamma m(t)}} dm(t) = \lambda\beta t^{\beta-1} dt \\
&\Rightarrow e^{-\gamma m(t)} dm(t) = \lambda\beta t^{\beta-1} dt \\
&\Rightarrow \int e^{-\gamma m(t)} dm(t) = \int \lambda\beta t^{\beta-1} dt \\
&\Rightarrow m(t) = -\frac{1}{\gamma}\ln(C-\gamma\lambda t^{\beta})
\end{aligned} \quad (9)$$

Using the initial condition $m(0)=0$, we can get $C=1$. Therefore

$$m(t) = -\frac{1}{\gamma}\ln(1-\gamma\lambda t^{\beta}) \quad (10)$$

and

$$\lambda(t) = \frac{\lambda\beta t^{\beta-1}}{1-\gamma\lambda t^{\beta}} \quad (11)$$

where $\gamma \neq 0$. When $\gamma = 0$, $\lambda(t) = \lambda\beta t^{\beta-1}$ and

$$m(t) = \int_0^t \lambda\beta t^{\beta-1} dt = \lambda t^{\beta} \quad (12)$$

which is the Crow-AMSAA model. From Eq. (10), it can be seen that $1-\gamma\lambda t^{\beta}$ should be greater than 0, therefore

$$\gamma < \frac{1}{\lambda t^{\beta}} \quad (13)$$

Since $\lim_{t\to\infty} \frac{1}{\lambda t^{\beta}} = 0$, in order to make Eq.(10) hold for any time, γ is set to be less than 0 in this paper, which means that the situation that the repair will worse the system is not considered. In fact, if the repairs increase the failure intensity of a system, there is no way that the system can survive for ever. For any model, long term prediction is not accurate because the system failure mechanism will change with time. Similarly, in this paper, the proposed model is only applied for short term prediction based on each particular application.

3. *PARAMETER ESTIMATION AND CONFIDENCE INTERVALS*

Maximal Likelihood Estimation (MLE) is used to estimate the model parameters. Let $t_1 < t_2 < \cdots < t_n$ denote the n failure times. In order to obtain the ML estimators of the parameters, consider the following definition of conditional probability:

$$F(t_i \mid t_{i-1}) = P(t \le t_i \mid t > t_{i-1}) = \frac{F(t_i) - F(t_{i-1})}{R(t_{i-1})} = 1 - \frac{R(t_i)}{R(t_{i-1})}$$

Using the empirical failure intensity in Eq. (4), The conditional reliability just before the i th failure will be:

$$R(t_i \mid t_{i-1}) = 1 - F(t_i \mid t_{i-1}) = e^{-\int_{t_{i-1}}^{t_i} \lambda(t)dt} = e^{-e^{(i-1)\gamma}\lambda(t_i^\beta - t_{i-1}^\beta)} \quad (14)$$

and the conditional p.d.f is

$$f(t_i \mid t_{i-1}) = \lambda\beta t_i^{\beta-1} e^{(i-1)\gamma} e^{-e^{(i-1)\gamma}\lambda(t_i^\beta - t_{i-1}^\beta)} \quad (15)$$

If the data is failure truncated the likelihood function will be:

$$L(\lambda,\beta,\gamma \mid Data) = f(t_n \mid t_{n-1}) f(t_{n-1} \mid t_{n-2}) \cdots f(t_1 \mid t_0) \quad (16a)$$

If the data is time truncated

$$L(\lambda,\beta,\gamma \mid Data) = f(t_n \mid t_{n-1}) f(t_{n-1} \mid t_{n-2}) \cdots f(t_1 \mid t_0) R(T \mid t_n) \quad (16b)$$

where T is the ending time of the test or observation. Clearly, Eq. (16a) is a special case of Eq. (16b) because when $T = t_n$, $R(T \mid t_n) = 1$.

Taking the natural log on both sides of Eq. (16b)

$$\ln(L) = n\ln\lambda + n\ln\beta + (\beta-1)\sum_{i=1}^{n}\ln t_i + \frac{n(n-1)}{2}\gamma - \lambda\sum_{i=1}^{n}\left(e^{(i-1)\gamma}t_i^\beta\right) + \lambda\sum_{i=1}^{n+1}\left(e^{(i-1)\gamma}t_{i-1}^\beta\right) - \lambda T^\beta e^{n\gamma} \quad (17)$$

By solving the non-linear equations of the first order derivatives with respect to the model parameters of the log likelihood function, the MLE estimates $\hat{\lambda}, \hat{\beta}, \hat{\gamma}$ can be obtained.

Confidence intervals on λ, β, γ can be obtained using asymptotic results. In practice, either the asymptotic normal distribution or the asymptotic lognormal distribution of the ML estimator is used. The approximation standard deviation of the ML estimators can be obtained from the estimated local Fisher's information matrix, which is:

$$\begin{bmatrix} \mathrm{var}(\lambda) & \mathrm{cov}(\lambda,\beta) & \mathrm{cov}(\lambda,\gamma) \\ \mathrm{cov}(\lambda,\beta) & \mathrm{var}(\beta) & \mathrm{cov}(\beta,\gamma) \\ \mathrm{cov}(\lambda,\gamma) & \mathrm{cov}(\beta,\gamma) & \mathrm{var}(\gamma) \end{bmatrix} = \begin{bmatrix} -\frac{\partial^2\ln(L)}{\partial\lambda^2} & -\frac{\partial^2\ln(L)}{\partial\lambda\partial\beta} & -\frac{\partial^2\ln(L)}{\partial\lambda\partial\gamma} \\ -\frac{\partial^2\ln(L)}{\partial\lambda\partial\beta} & -\frac{\partial^2\ln(L)}{\partial\beta^2} & -\frac{\partial^2\ln(L)}{\partial\gamma\partial\beta} \\ -\frac{\partial^2\ln(L)}{\partial\lambda\partial\gamma} & -\frac{\partial^2\ln(L)}{\partial\gamma\partial\beta} & -\frac{\partial^2\ln(L)}{\partial\gamma^2} \end{bmatrix}^{-1}_{\lambda=\hat{\lambda};\beta=\hat{\beta};\gamma=\hat{\gamma}} \quad (18)$$

The entries in Eq. (18) is

$$\frac{\partial^2\ln(L)}{\partial\lambda^2} = -\frac{n}{\lambda^2}$$

$$\frac{\partial^2\ln(L)}{\partial\beta^2} = -\frac{n}{\beta^2} - \lambda\sum_{i=1}^{n}\left\{e^{(i-1)\gamma}t_i^\beta\left(\ln t_i\right)^2\right\} + \lambda\sum_{i=1}^{n}\left\{e^{(i-1)\gamma}t_{i-1}^\beta\left(\ln t_{i-1}\right)^2\right\}$$

$$\frac{\partial^2\ln(L)}{\partial\gamma^2} = -\lambda\sum_{i=1}^{n}\left\{e^{(i-1)\gamma}t_i^\beta(i-1)^2\right\} + \lambda\sum_{i=1}^{n}\left\{e^{(i-1)\gamma}t_{i-1}^\beta(i-1)^2\right\}$$

$$\frac{\partial^2\ln(L)}{\partial\lambda\partial\beta} = -\sum_{i=1}^{n}\left\{e^{(i-1)\gamma}\left(t_i^\beta\ln t_i - t_{i-1}^\beta\ln t_{i-1}\right)\right\}$$

$$\frac{\partial^2\ln(L)}{\partial\lambda\partial\gamma} = -\sum_{i=1}^{n}\left\{e^{(i-1)\gamma}\left(t_i^\beta - t_{i-1}^\beta\right)(i-1)\right\}$$

$$\frac{\partial^2\ln(L)}{\partial\gamma\partial\beta} = -\lambda\sum_{i=1}^{n}\left\{e^{(i-1)\gamma}t_i^\beta\ln t_i\cdot(i-1)\right\} + \lambda\sum_{i=1}^{n}\left\{e^{(i-1)\gamma}t_{i-1}^\beta\ln t_{i-1}\cdot(i-1)\right\}$$

From Eq. (18), the standard deviation and covariance of the model parameters can be calculated. For example, the standard deviation of parameter λ is

$$\sigma_{\hat{\lambda}} = \sqrt{\mathrm{var}(\hat{\lambda})}$$

Depending on the asymptotic distribution (normal or lognormal) of λ, the two sided confidence interval for λ is either

$$\hat{\lambda} \pm z_{\alpha/2}\sigma_{\hat{\lambda}} \qquad \text{or} \qquad \hat{\lambda}\exp(\pm z_{\alpha/2}\sigma_{\hat{\lambda}}/\hat{\lambda})$$

where $z_{\alpha/2}$ is the $\alpha/2$ quantile of the standard normal distribution (Ref. 10). The confidence interval of other model parameters also can be calculated using the same method.

Bounds on Expected Failure Number $m(t)$ and $\lambda(t)$

The bounds of the expected failure number and instantaneous failure intensity can be calculated based on the variance and covariance of the model parameters. For example, in order to get the bounds of $m(t)$, the variance of $m(t)$ can be calculated using the following procedure. First, take the derivatives of $m(t)$ respect to each model parameter:

$$\frac{dm(t)}{d\lambda} = -\frac{1}{\gamma}\frac{-\gamma t^\beta}{1-\gamma\lambda t^\beta} = \frac{t^\beta}{1-\gamma\lambda t^\beta}$$

$$\frac{dm(t)}{d\beta} = -\frac{1}{\gamma}\frac{-\gamma\lambda t^{\beta}\ln t}{1-\gamma\lambda t^{\beta}} = \frac{\lambda t^{\beta}\ln t}{1-\gamma\lambda t^{\beta}}$$

$$\frac{dm(t)}{d\gamma} = \frac{1}{\gamma^2}\ln(1-\gamma\lambda t^{\beta}) + \frac{1}{\gamma}\frac{\lambda t^{\beta}}{1-\gamma\lambda t^{\beta}}$$

Then, using the delta rule, the variance of $m(t)$ is

$$\begin{aligned}
\mathrm{var}(m(t)) = {} & \left(\frac{\partial m(t)}{\partial\beta}\right)^2 \mathrm{var}(\hat{\beta}) + \left(\frac{\partial m(t)}{\partial\lambda}\right)^2 \mathrm{var}(\hat{\lambda}) + \left(\frac{\partial m(t)}{\partial\gamma}\right)^2 \mathrm{var}(\hat{\gamma}) \\
& + 2\left(\frac{\partial m(t)}{\partial\beta}\right)\left(\frac{\partial m(t)}{\partial\lambda}\right)\mathrm{cov}(\hat{\beta},\hat{\lambda}) + 2\left(\frac{\partial m(t)}{\partial\gamma}\right)\left(\frac{\partial m(t)}{\partial\lambda}\right)\mathrm{cov}(\hat{\gamma},\hat{\lambda}) \\
& + 2\left(\frac{\partial m(t)}{\partial\beta}\right)\left(\frac{\partial m(t)}{\partial\gamma}\right)\mathrm{cov}(\hat{\beta},\hat{\gamma})
\end{aligned}$$

If $m(t)$ is assumed to be log normally distributed, the two sided confidence interval of $m(t)$ can be obtained as:

$$[m(t)_L,\ m(t)_U] = [e^{-z_{\alpha/2}\sigma_{\hat{m}(t)}/\hat{m}(t)},\ e^{z_{\alpha/2}\sigma_{\hat{m}(t)}/\hat{m}(t)}]$$

where, $\sigma_{\hat{m}(t)} = \sqrt{\mathrm{var}(\hat{m}(t))}$.

If $m(t)$ is assumed to be normally distributed the two sided confidence interval of $m(t)$ can be obtained as:

$$\begin{aligned}
& [m(t)_L,\ m(t)_U] \\
& = [\max(0, \hat{m}(t) - z_{\alpha/2}\sigma_{\hat{m}(t)}),\ \hat{m}(t) + z_{\alpha/2}\sigma_{\hat{m}(t)}]
\end{aligned}$$

Usually, in practice, $m(t)$ is assumed to be lognormally distributed since it should always be a positive number. The same procedure can be used to calculate the confidence interval of instantaneous failure intensity $\lambda(t)$ and instantaneous MTBF.

4. *NUMERICAL APPLICATIONS*

Two numerical applications are used to illustrates the proposed model and the related estimation. Comparisons with the Crow-AMSAA model are also given.

Automobile Data

First, the data set given by Ahn et al (Ref.11) and used by Pulcini (Ref. 3) is considered here. It is the subset #4 of the data related to six automobiles (1973 AMC Ambassador) once owned by the Ohio state government. In both papers, the Crow-AMSAA model is applied. It is also shown that there is a clear time trend of the failure intensity. This subset is given in Table 1, which consists of 18 times to failure (in days) for all causes of failure for that automobile. Times between failures can be obtained by successive subtractions.

Table 1: Times to Failure: Automobile Data

202	571	868	1108	1376
265	755	999	1230	1447
363	770	1054	1268	
508	818	1068	1330	

The results of the Crow-AMSAA model and the proposed model are given in Table 2.

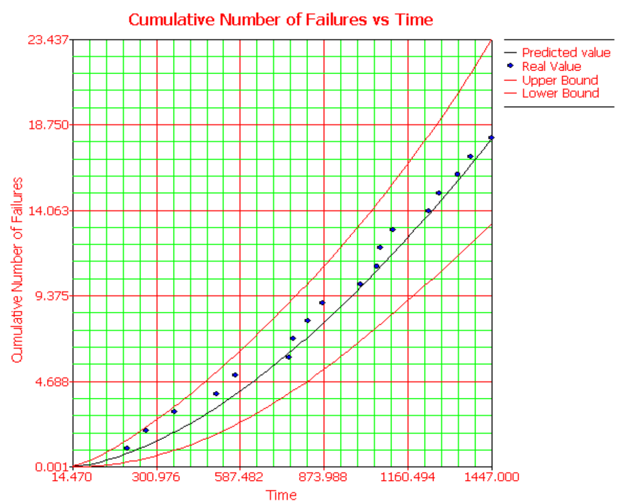
Table 2: Parameter Estimates: Automobile Data

Model Parameter	Crow-AMSAA	Proposed Model
β	1.62582719	2.47315266
λ	0.00013089	7.94E-07
γ	N/A	-0.10927712
Log-likelihood	-95.1471	-94.5997

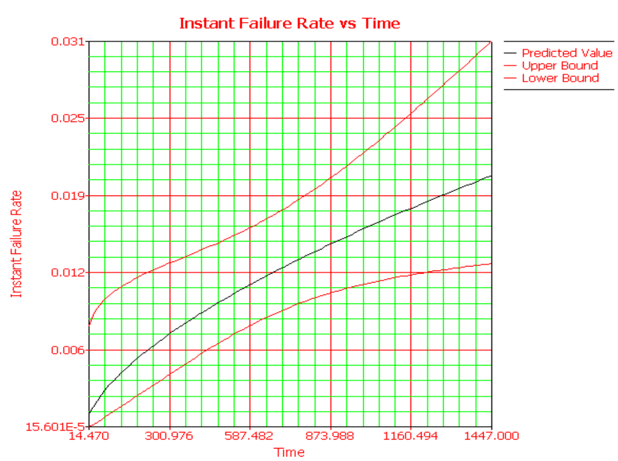
From Table 2, it can be found that the proposed model is better than the Crow-AMSAA model in terms of likelihood values. The repair effects are reflected by the parameter of γ. Both models show that there is an increasing failure intensity. The χ^2 test is used to test the significance of the additional parameter γ at a given confidence level. The χ^2 test can be expressed as:

$$-2(\ln L_0 - \ln L_1) \sim \chi_1^2$$

where $\ln L_0$ and $\ln L_1$ are the log-likelihood value for

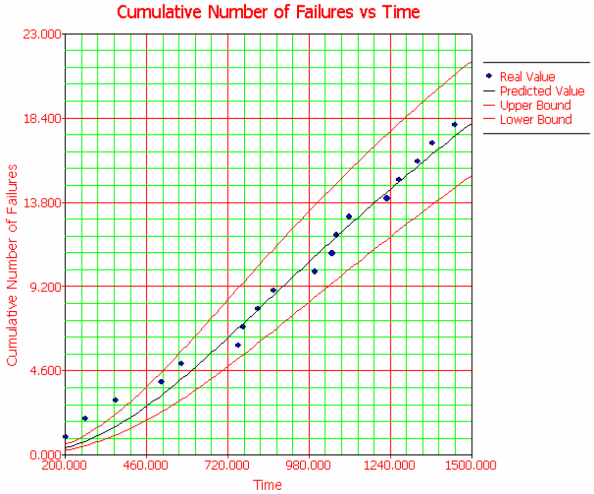


(a)

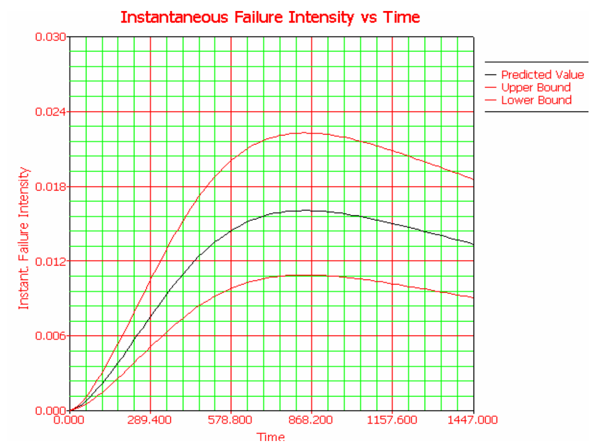


(b)

Figure 1: a) Failure Number and b) Instantaneous Failure Intensity, Crow-AMSAA model (Automobile Data)



(a)



(b)

Figure 2: a) Failure Number and b) Instantaneous Failure Intensity, Proposed model (Automobile Data)

AMSAA and the proposed model. Given $\alpha = 0.1$, the critical value $\chi^2_{0.1,\,1} = 2.71$ and $-2(\ln L_0 - \ln L_1) = 1.0948$. So, the effect of γ is not significant at significant level 0.1.

The predicted failure number and instantaneous failure intensity with their 95% confidence bounds for both models are given in Figure 1 and 2.

Comparing Figure 1 with Figure 2, we can find that both models can predict the failure number very well. However, the prediction of the instantaneous failure intensity is different. In Figure 2, the failure intensity increases first, and then decreases. The increase could be caused by the fact that more and more functions and components are tested and used. Then the rate of uncovering latent failures increases. The decrease is caused by the repair activities.

Aircraft Air-conditioning Equipment Data

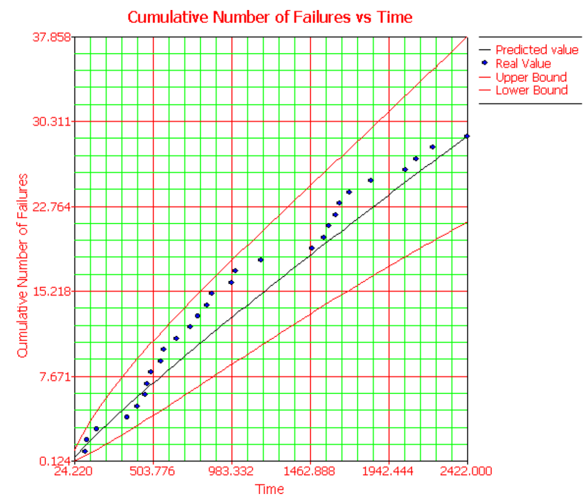
A subset of the failure data of the 13 Boeing aircraft given by Proschan (Ref. 12) is considered here. The subset noted as Aircraft #3, which consists of 29 times to failure (in hours) is reported in Table 3.

Table 3: Times to Failure: Aircraft Air-Condition Data

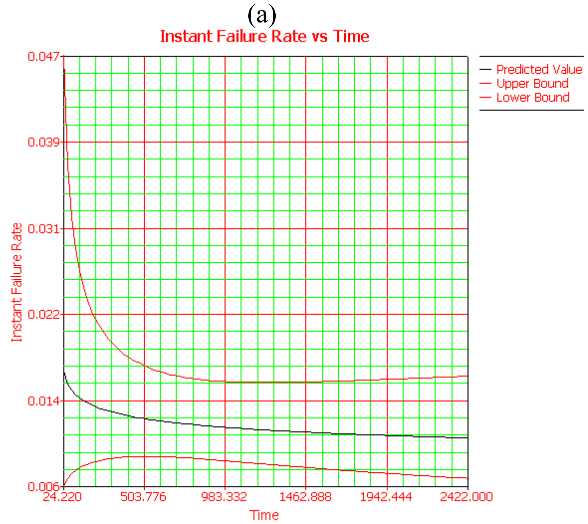
90	470	777	1474	1835
100	494	836	1550	2043
160	550	865	1576	2113
346	570	983	1620	2214
407	649	1008	1643	2422
456	733	1164	1705	

Table 4: Parameter Estimates: Aircraft Air-Condition Data

Model Parameter	Crow-AMSAA	Proposed Mode
β	0.90073	2.0381
λ	0.0260	4.09E-05
γ	N/A	-0.1344
Log-likelihood	-157.1624	-154.7828



(a)

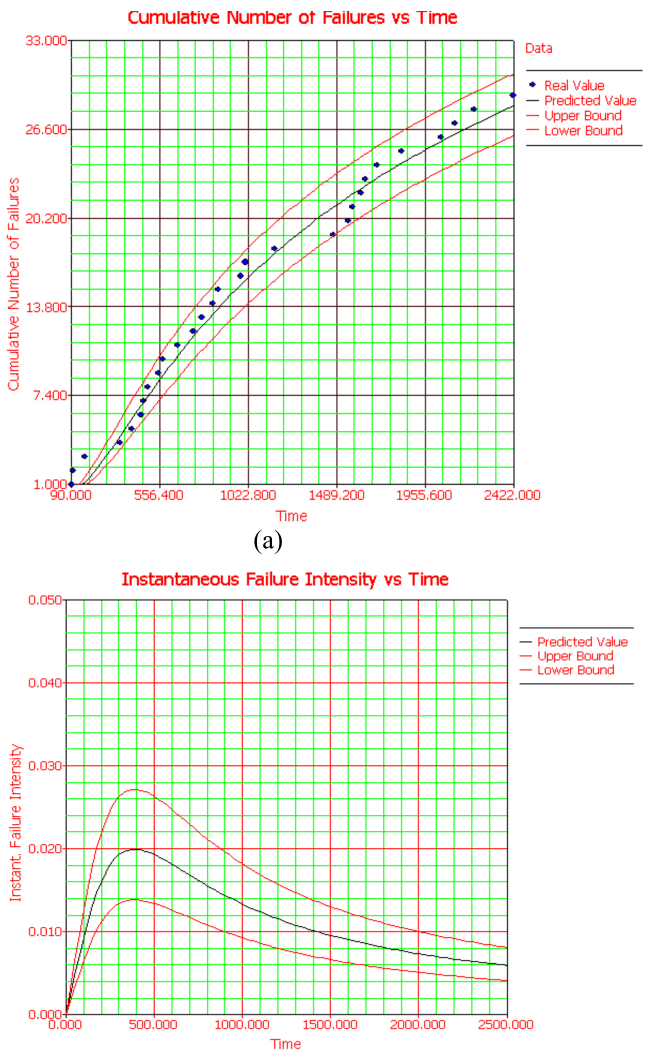


(b)

Figure 3: a) Failure Number and b) Instantaneous Failure Intensity, Crow-AMSAA model (Aircraft Air-Condition Data)

The results of the Crow-AMSAA model and the proposed model are given in Table 4 and Figures 3 and 4.

From the above Table, we know that $-2(\ln L_0 - \ln L_1) = 4.7592$, which is greater that the critical chi-squared value $\chi^2_{0.1,\,1} = 2.71$. The effect of the third parameter is significant at significant level 0.1.



(a) (b)

Figure 4: a) Failure Number and b) Instantaneous Failure Intensity, Proposed model (Aircraft Air-Condition Data)

From Table 4, based on LKV, the proposed model is better than the Crow-AMSAA model. From Figure 4, it can be seen that the system goes to a period of decreasing failure intensity after experiencing an increasing period.

From both case studies, we can see that the proposed model can fit repairable system data very well and the repair effects are reflected by introducing one more model parameter.

5. *CONCLUSION AND FUTURE WORK*

In this paper, we reviewed two popular used models for repairable systems. In order to overcome the drawbacks of these models, a new model which considers both time trend and repair effects was proposed. The proposed model has clear physical meaning and can be easily used to solve real engineering problems in practice. The ML estimators for the NHPP and the proposed model were provided. The confidence bounds for reliability metrics such as cumulative failure number and instantaneous failure intensity were also provided. The application of the proposed method was illustrated by two real case studies. Comparison results show that the proposed model is very useful in practice.

In this paper, the power law function was used as the baseline function of the failure intensity. Functions with other forms such as log linear, linear, and exponential can also be used in further studies. How to extend the proposed model to multiple systems, grouped data will be studied in further research.

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