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Reliability Prediction using Multivariate Degradation Data

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Key Words: Degradation Testing, State-Space Model, Logistic Regression, Multiple Degradation Measure

SUMMARY & CONCLUSIONS

This paper presents a general degradation modeling and analysis approach involving multiple degradation measures. We first explore the correlation between the degradation measure and the failure event by introducing a probabilistic measure. A logistic function is used to define this probabilistic measure and assess the likelihood of a failure event given the degradation level. When the stochastic stress condition is considered this probabilistic measure is an approximation of the likelihood of a failure event. We then propose a state-space model to describe the evolution of the degradation process by incorporating both the degradation dynamics and random stress effects. Finally the degradation dynamics is used to predict the reliability function. The practical use of the proposed method is demonstrated by a case study.

1. *INTRODUCTION*

The advances in materials science, manufacturing processes and quality/reliability engineering have led to few or no hard failures at normal conditions, or even at accelerated conditions, thus it is difficult to obtain the accurate reliability estimation based on life data. Reliability prediction based on degradation modeling can be an efficient and alternative method to estimate reliability for some highly reliable parts or systems. As the important performance parameter gradually degrades to a critical threshold level, systems and its components are defined as degradation (soft) failure. Many products exhibit this failure mode, such as semiconductors, mechanical systems and microelectronics.

In recent years, the studies of performance degradation have attracted many interests and efforts because the degradation measurements contain fairly credible, accurate and useful information about product reliability. Nelson (1990) briefly surveys the degradation behavior of various products and materials subject to degradation, accelerated degradation models and inference procedures. Lu and Meeker (1993), Tseng, Hamada and Chiao (1994), and Meeker, Escobar and Lu (1998) considered *general degradation path models*. Suzuki, Maki and Yokogawa (1993) used *linear degradation models* to study the increase in a resistance measurement over time. Meeker and Escobar (1998) used *concave degradation models* to study the growth of failure-causing conducting filaments of chlorine-copper compound in printed-circuit boards. Carey and Koenig (1991) used similar models to describe degradation of electronic components. Yang and Xue (1996) presented *random process* to model performance degradation and estimated the reliability of products. Eghbali (1999) developed the *degradation hazard function approach* for the analysis of degradation data. Zhao and Elsayed (2004) proposed the *Inverse-Power-Law-Weibull-Brownian* model for analyzing competing risk data involving performance degradation and hard failures obtained at accelerated operating conditions. Wang and Coit (2004) describes system-based degradation model. However, most of these previous research has been focusing on reliability prediction based on single degradation measure or component failure mechanism level. In practice, a system may consist of multiple components or a component may have multiple degradation measures, so it is necessary to simultaneously consider multiple degradation measures.

In this paper we first explore the correlation between the degradation measure and the failure event by introducing a probabilistic measure. We use a logistic function to define this probabilistic measure, which is used to assess the likelihood of a failure event given the degradation level. We argue this probabilistic measure is an approximation of the likelihood of a failure event when the stochastic stress condition is considered. We use a state-space model to describe the evolution of the degradation process by incorporating both the degradation dynamics and random stress effects. Finally the degradation dynamics is used to predict the reliability function.

2. *PROBABILITY OF A FAILURE EVENT GIVEN DEGRADATION LEVELS*

Assume the vector $\boldsymbol{x}$ contains m degradation measures ($x_1, x_2, \cdots x_m$). The standard definition of a degradation failure event is $\bigcup_{i=1}^{m}\{x_i > d_i\}$ where d_i is the critical level for the i^{th} degradation measure. One implication of this definition is given the degradation levels in $\boldsymbol{x}$, we know exactly whether a failure occurs or not. It's a black-and-white definition without reference to how likely a failure will happen given the degradation levels. Below we use a logistic function to define the probability of a failure event given the values for the

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degradation measures x

$$Pr\{failure \mid x\} = \frac{e^{\beta' x}}{1 + e^{\beta' x}} = \frac{exp(\sum_{i=1}^{m} \beta_i x_i)}{1 + exp(\sum_{i=1}^{m} \beta_i x_i)} \quad (1)$$

The reason to choose a logistic function in eqn. (1) is it's a standard statistical technique to predict the distribution of a binary variable given predictors in x. For example, given the degradation level such as the crack size on a bridge, the certainty of a catastrophic failure such as the collapse of the bridge, is better measured by a probabilistic measure between 0 and 1. This probability measure is not only related to the level of the crack size, it is a function of the environmental stress as well. The larger the stress (e.g., the load the bridge is supporting), the more likely a hard failure will occur.

Given the sample data with pairs of the observed degradation measures and the response value of whether a hard failure occurs, we can estimate the coefficients β in eqn. (1) using Fisher scoring algorithm, see Dobson (1990).

Example 1: The following data set is obtained from the degradation testing at 678 hours. There are two degradation measures for each test unit. The fourth column is status corresponding to catastrophic failure, we assume the catastrophic failure is due to these two degradation measuers. We can use Eqn. (1) to estimate the probability of a failure event or reliability given these two degradation measures.

Table 1. Degradation Data Set

Units	Degradation 1	Degradation 2	S/F
1	0.0333	0.0217	S
2	0.0233	0.0345	F
3	0.0137	0.0454	F
4	0.0156	0.0385	S
5	0.0159	0.0392	F
6	0.0286	0.0263	S
7	0.0333	0.0313	F
8	0.0385	0.0357	F
9	0.0270	0.0354	F
10	0.0156	0.0294	S
11	0.0230	0.0353	F
12	0.0222	0.0387	S
13	0.025	0.0454	F
14	0.0234	0.0294	F
15	0.0210	0.0333	S
16	0.0217	0.0355	F

Fot this particular problem, we obtain β_0 = -16.85, β_1 = 245.3 and β_2 = 343.2. Suppose customers are interested in the reliability of this product at time equal to 1000 hours, first we use the traditional model to obtain the degradation level at time 1000 hours, x_1=0.0265, x_2=0.0377, thus we can use Eqn. (1) to estimate the probability of a failure event or reliability at time 1000 hours.

$$Pr\{failure \mid x_1 = 0.0265, x_2 = 0.0377\}$$
$$= \frac{\exp(-16.85 + 245.3 \times 0.0265 + 343.2 \times 0.0377)}{1 + \exp(-16.85 + 245.3 \times 0.0265 + 343.2 \times 0.0377)}$$
$$= 0.93$$

3. *PROBABILITY OF A FAILURE EVENT CONSIDERING THE RANDOM STRESS CONDITIONS*

The previous section defines the probabilistic measure to indicate the likelihood of a failure given the degradation levels $\boldsymbol{x}$. This section gives a generic description of this measure by considering the stochastic nature of the stresses applied to the system. This description is also related to the usual definition of a failure event using criterion whether the degradation level exceeds the critical value d.

The following picture in Figure 1 illustrates the structure of a system we will consider throughout this paper. Both degradation levels and random stresses in Figure 1 affect how a system is successfully running. The stress levels, at the same time, will also affect how the degradation process evolves.

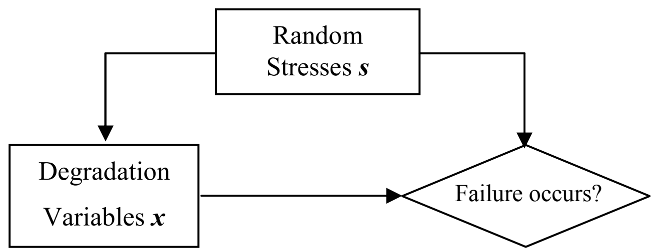

Figure 1. Both degradation process and random stress affect the likelihood of the occurrence of a failure.

Using the vector s to denote the random stress variables, we can define a failure event in the following way:

$$\{\text{failure}\} = \{ g(x, s) > 0 \} \quad (2)$$

One example of $g(x, s)$ using the definition of critical levels d is

$$g(x, s) = Max_{i=1}^{m} \{x_i - d_i(s)\} \quad (3)$$

Eqn. (3) implies if the function $g(x, s) > 0$, then there exists a degradation variable r for which $x_r > d_r(s)$, which implies the occurrence of the failure. The critical level d_i in eqn. (3) is a decreasing function of the stress level s. It indicates the higher the stress level, the lower the threshold so that the degradation process is more likely to cause the failure.

Using the failure definition in eqn. (2), we can calculate the probability of a failure event given the degradation levels in $\boldsymbol{x}$ as follows

$$Pr\{failure \mid x\} = \int_s I_{\{g(x,s)>0\}}(s) f_S(s) ds \quad (4)$$

where $I(s)$ is an indicator function and $f_S(s)$ is the *pdf* of the stress variables s.

4. *DYNAMICS OF DEGRADATION PROCESS*

We use a state-space model to describe the degradation dynamics. Figure 1 indicates the degradation rate not only depends on the current degradation levels, it also has to do with the stress levels. For example, consider the crack size in a bridge, the current size of the crack determines how fast a crack grows over the next period, it's why a crack grows very slowly in the initial stage while it grows much faster when the crack is visible to the eyes. On the other hand, the growth rate also depends on various stress levels such as how much load the bridge supports and how cold the temperature is in the winter.

The following state-space model is used to describe the dynamics of the degradation process.

$$\frac{dx}{dt} = H(x,s) \tag{5}$$

where $H(x,s)$ is an increasing function of both degradation variables *x* and stress variables *s*.

Although $H(x,s)$ can be a nonlinear function, this paper only considers the linear version of eqn (5):

$$\frac{dx}{dt} = Fx + Gs \tag{6}$$

where F and G are two matrices and the random stress variables $\boldsymbol{s}$ is normally distributed with mean μ_s and variance Σ_s. We further assume the stress variables are uncorrelated between any two different time points, *i.e.*, $Cov(s_i, s_j) = 0$ for $i \neq j$, therefore $s - \mu_s$ is multivariate white noise. We further assume the degradation process starts with the initial state x_0.

Eqn. (6) is equivalent to the following form

$$dx = Fxdt + Gsdt \tag{7}$$

We know from stochastic analysis that white noise is the derivative of a Brownian motion process, i.e., $dW = (s - \mu_s)dt$, where W_t is a Brownian motion with $Var(W_t) = \Sigma_s t$. Therefore eqn. (7) can be written as

$$dx = Fxdt + G\mu_s dt + GdW \tag{8}$$

The discrete-time version of eqn. (8) is

$$x_n = x_{n-1} + Fx_{n-1}\delta t + G\mu_s \delta t + G(W_n - W_{n-1}) \tag{9}$$

where δt is the time interval between two consecutive time points. It's easy to show that

$$x_n = (I + F\delta t)^n x_0 + \sum_{i=0}^{n-1} (I + F\delta t)^i G\mu_s \delta t + \sum_{i=0}^{n-1} (I + F\delta t)^{n-1-i} G(W_{i+1} - W_i) \tag{10}$$

Therefore the mean of x_n is

$$\mu_n = E(x_n) = (I + F\delta t)^n x_0 + \sum_{i=0}^{n-1} (I + F\delta t)^i G\mu_s \delta t \tag{11}$$

The continuous version of eqn. (8) can be derived by forcing $n \to \infty$ and $n\delta t \to t$. Using the limit of matrix power, we have

$$(I + F\delta t)^n \to e^{Ft}. \tag{12}$$

To derive $\sum_{i=0}^{n-1} (I + F\delta t)^i \delta t$, let $Z_n = \sum_{i=0}^{n-1} (I + F\delta t)^i \delta t$, thus $Z_n = Z_{n-1} + (I + F\delta t)^{n-1}\delta t$, or $\frac{Z_n - Z_{n-1}}{\delta t} = (I + F\delta t)^{n-1}$. Let $\delta t \to 0$, we have $\frac{dZ}{dt} = e^{Ft}$, therefore

$$\sum_{i=0}^{n-1} (I + F\delta t)^i \delta t = Z_n \to Z = VW^{-1}U'e^{Ft} \tag{13}$$

where we assume the matrix F has the singular value decomposition (SVD): $F = UWV'$ (U, V are orthonormal matrices satisfying $UU' = VV' = I$ and W is a diagonal matrix). Substituting eqns (12) and (13) into (10), we get

$$\mu_t = e^{Ft} x_0 + VW^{-1}U'e^{Ft} G\mu_s \tag{14}$$

We now derive the variance of x_n

$$\Sigma_n = Var(x_n) = Var\left[\sum_{i=0}^{n-1} (I + F\delta t)^{n-1-i} G(W_{i+1} - W_i)\right]$$

$$= Var\left[G(W_n - W_{n-1}) + (I + F\delta t)\sum_{i=0}^{n-2} (I + F\delta t)^{n-2-i} G(W_{i+1} - W_i)\right]$$

$$= G\Sigma_s G'\delta t + (I + F\delta t)\Sigma_{n-1}(I + F'\delta t)$$

$$= \Sigma_{n-1} + (G\Sigma_s G' + F\Sigma_{n-1} + \Sigma_{n-1}F')\delta t + F\Sigma_{n-1}F'(\delta t)^2$$

Let $n \to \infty$ and $n\delta t \to t$, we have

$$\frac{d\Sigma_t}{dt} = F\Sigma_t + \Sigma_t F' + G\Sigma_s G' \tag{15}$$

We note eqn. (15) is a matrix Riccati equation. When the degradation process is in one dimension, eqn. (15) becomes

$$\frac{d\Sigma_t}{dt} = 2F\Sigma_t + G\Sigma_s G' \tag{16}$$

where both Σ_t and F are scalars

5. *RELIABILITY PREDICTION AND NUMBERICAL EXAMPLES*

The methodology from the previous section was applied to experimental data in this section where we analyze an accelerated testing experiment conducted in the Quality and Reliability Engineering Laboratory at Rutgers University. The purpose of this experiment is to study the effect of stress on light emitting diodes (LEDs) and to predict their reliability under operating conditions.

The reliability of LEDs is strongly dependent on the degradation mode and device characteristics such as current versus optical output power and operating temperature. The influence of physical degradation on the degradation rate of the device characteristics is affected by the device characteristics themselves. The correlation between reliability and degradation modes is not so common. LED degradation modes are studied and rapid degradation is found to be related to the generation or growth of dark spot/line defects. At higher current density, voltage, or temperature, rapid power reduction due to dark spot/line defect generation occurs. Two primary causes for the dark spot/line defects are identified. They are precipitation of host atoms and the migration of electrode metal into the semiconductor (Fukuda, Fujita, and Iwane, 1983).

In our experiment, we assume that an LED fails when its performance reaches a specified rapid degradation level (degradation failure) that is defined by an additional test. To continuously record the failure times of testing units and to control the applied factors, an automatic accelerated testing environment is designed. Figure 2 depicts the layout of the experimental equipment.

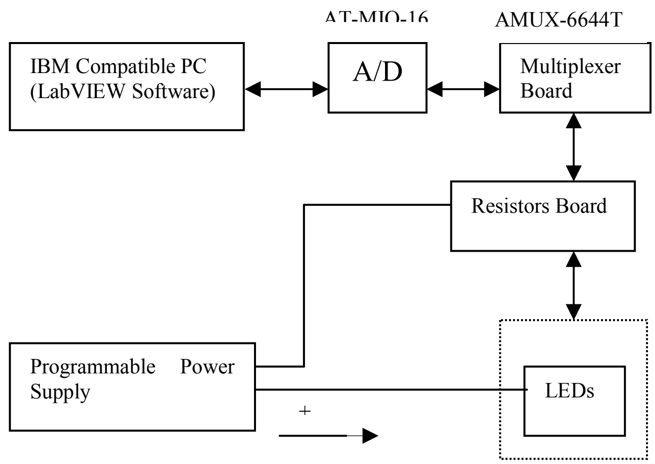

Figure 2. The Layout of Experimental Equipment

Figure 3 is a circuit diagram for testing the LEDs. It consists of a power supply, adjustable resistor and the environmental stress conditions. The adjustable resistor is connected with LEDs. The voltage of the resistor is monitored in order to determine the current of the LEDs. It should be pointed out that it is assumed that the resistance of the adjustable resistor does not change with time.

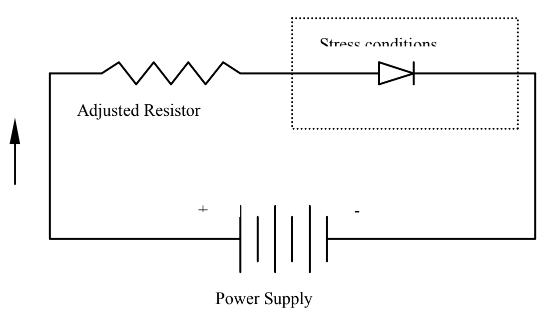

Figure 3. Adjustable Resistor and LEDs Circuit

The experiment is conducted at three different stress levels 40mA, 35mA, and 28mA. We use data obtained from stress levels 40mA, 35mA to estimate the model, and then validate the model using 28mA data. At each stress levels we conduct six accelerated life testing experiments, for each experiment there are 32 samples for testing, so basically there are 192 samples for testing at each stress level; In each test a designed-circuit-board that contains 32 randomly chosen LEDs is placed in a temperature chamber where the temperature and current in the circuit are held constant. The light intensity of LEDs is then measured at room temperature every 50 hours. We utilize the original decreasing degradation (light intensity of LEDs) paths as degradation measure. According to Eqn. (14), we can obtained the following table for degradation measure estimation at specific time t and different stress level.

Table 2 Degradation (LEDs Light Intensity) Estimation

T	0	50	100	150	200	250
40mA	100.2301	78.0592	60.7926	47.3453	36.8725	28.7164
35mA	101.8401	79.3131	61.7691	48.1058	37.4648	29.1777
28mA	103.4501	80.5670	62.7456	48.8663	38.0571	29.6390

Using Eqn (15), we obtain the variance of degradation measure which is 3.98, and we define that the degradation critical level is 5.3, thus we can use eqn (4) to calculate the failure probability and reliability. Figure 4 is the reliability estimation for LEDs at 28mA.

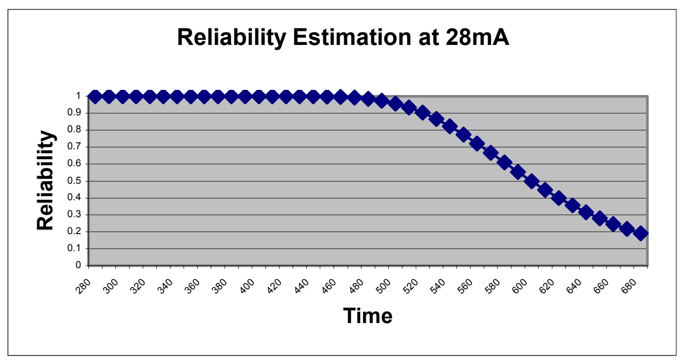

Figure 3 Reliability Estimation at 28mA

6. *CONCLUSIONS*

This paper introduces two techniques to model and analyze system with multiple degradation measures. We first explores the correlation between the degradation measure and the failure event by introducing a probabilistic measure. A logistic function is used to define this probabilistic measure and assess the likelihood of a failure event given the degradation level. When the stochastic stress condition is considered this probabilistic measure is an approximation of the likelihood of a failure event. We then propose a state-space model to describe the evolution of the degradation process by incorporating both the degradation dynamics and random stress effects. Finally the degradation dynamics is used to predict the reliability function.

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