System Reliability Analysis: The Advantages of Using Analytical Methods to Analyze Non-Repairable Systems

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SUMMARY & CONCLUSIONS

Most of the system analysis software available on the market today employs the use of simulation methods for estimating the reliability of non-repairable systems. Even though simulation methods are easy to apply and offer great versatility in modeling and analyzing complex systems, there are some limitations to their effectiveness. For example, if the number of simulations performed is not large enough, these methods can be error prone. In addition, performing a large number of simulations can be extremely time-consuming and simulation offers a small range of calculation results when compared to analytical methods. Analytical methods have been avoided due to their complexity in favor of the simplicity of using simulation. A software tool has been developed that calculates the exact analytical solution for the reliability of a system. Given the reliability equation for the system, further analyses on the system can be performed, such as computing exact values of the reliability, failure rate, at specific points in time, as well as computing the system MTTF (mean time to failure), and reliability importance measures for the components of the system. In addition, optimization and reliability allocation techniques can be utilized to aid engineers in their design improvement efforts. Finally, the time-consuming calculations and the non-repeatability issue of the simulation methodology are eliminated.

1. INTRODUCTION

The analytical approach involves the determination of a mathematical expression which describes the reliability of the system, expressed in terms of the reliabilities of its components. For example, in the case of a system with three statistically independent components in series, the system's reliability equation is given by,

$$R_S(t) = R_1(t) \cdot R_2(t) \cdot R_3(t). \tag{1}$$

The reliability of the system for any mission time can now be estimated. Assuming a Weibull life distribution for each component, eq (1) can now be expressed in terms of each component's reliability function, or

$$R_S(t) = e^{-\left(\frac{t}{\eta_1}\right)^{\beta_1}} \cdot e^{-\left(\frac{t}{\eta_2}\right)^{\beta_2}} \cdot e^{-\left(\frac{t}{\eta_3}\right)^{\beta_3}}.$$

In the same manner, any life distribution can be substituted into the system reliability equation. Suppose that the times-to-failure of the first component are described with a Weibull distribution, the times-to-failure of the second component with an exponential distribution, and the times-to-failure of the third component with a normal distribution. Then, eq (1) can be written as,

$$R_S(t) = e^{-\left(\frac{t}{\eta_1}\right)^{\beta_1}} \cdot e^{-\lambda_2 \cdot t} \cdot \left[1 - \Phi\left(\frac{t - \mu_3}{\sigma_3}\right)\right].$$

It can be seen that the biggest challenge is in obtaining the system's reliability function in terms of component reliabilities. Once this has been achieved, calculating the reliability of the system for any mission duration is just a matter of substituting the corresponding component reliability functions into the system reliability equation. The more complex the system is, the harder and more time consuming it becomes to obtain an analytical solution for the system. The simplest and most popular way of solving such complex systems is through simulation. Even though simulation is a great tool, and offers solutions where everything else fails, it does have its limitations in terms of time, repeatability and additional analysis options.

On the other hand, the primary advantage of the analytical solution is that a mathematical expression which describes the reliability of the system is obtained. Once the system's reliability function has been determined, other calculations on the system can then be performed. Such calculations include:

1. Determination of the system's *pdf*.
2. Determination of warranty periods.
3. Determination of the system's failure rate.
4. Determination of the system's MTTF.
5. Reliability importance of components.
6. Utilization of optimization and reliability allocation techniques.
7. Analysis of systems consisting of a mixture of static and time-dependent components.

In this paper the advantages of using analytical solutions in system analysis will be illustrated. The term “analytical solution” actually refers to a numerical solution, since results will be obtained with the use of software.

2. NOTATION

b	Weibull shape parameter,
h	Weibull scale parameter,
C	total system cost,
c_i	cost of component/subsystem i,
$f_S(t)$	system probability density function,
I_R	reliability importance,
$l_S(t)$	system failure rate,
MTTF	Mean Time To Failure,
n	number of components within the system considered in the optimization,
pdf	probability density function,
$R_{i,min}$	minimum reliability of component/subsystem i
$R_{i,max}$	maximum achievable reliability of component/subsystem i,
R_i	reliability of component/subsystem i,
R_s	system reliability,
R_G	system reliability goal

3. ASSUMPTIONS

1. All systems consist of s-independent components/subsystems.
2. The system and its components/subsystems can only assume two states, failed and operational.
3. In repairable systems only the failure properties of the components/subsystems are considered.

4. BACKGROUND THEORY

4.1 *System Probability Density Function (pdf)*

Once the equation for the reliability of the system is obtained, the system's *pdf* can be determined. The *pdf* is the derivative of the reliability function with respect to time:

$$f_S(t) = -\frac{d\left(R_S(t)\right)}{dt}. \tag{2}$$

4.2 *System Failure Rate*

The failure rate can also be obtained by dividing the *pdf* by the reliability function:

$$l_S(t) = \frac{f_S(t)}{R_S(t)} \tag{3}$$

4.3 *System Mean Time to Failure*

The mean time-to-failure, MTTF, can obtained by integrating the system reliability function from zero to infinity:

$$MTTF = \int_0^\infty R_S(t)dt. \tag{4}$$

4.4 *Warranty Period*

Sometimes, it is desirable to know the time value associated with a certain reliability. Warranty periods are often calculated by determining what percentage of the failure population can be financially covered, and estimating the time at which this portion of the population will fail. Similarly, engineering specifications may call for a certain BX life, which also represents a time period during which a certain proportion of the population will fail. For example, the B10 life is the time in which 10% of the population will fail. These calculations can be achieved by solving the system reliability equation for the time, given an associated reliability (or unreliability). To illustrate, consider a system of three statistically independent units connected reliability-wise in series. For the first unit, the times-to-failure are modeled with a Weibull distribution, for the second unit an exponential distribution, and for the third unit a normal distribution. The system reliability can then be expressed as,

$$R_S(t) = e^{-\left(\frac{t}{h_1}\right)^{b_1}} \cdot e^{-l_2 \cdot t} \cdot \left[1-\Phi\left(\frac{t-m_3}{s_3}\right)\right]. \tag{5}$$

In order to find the warranty period, eq (5) must be solved for the time, t, given an acceptable system reliability value, R_S. In this example, a closed form solution cannot be obtained and it is necessary to solve for t numerically.

4.5 *Conditional Reliability*

Conditional reliability is the probability of a system successfully completing another mission following the successful completion of a previous mission. The time of the previous mission as well as the time for the mission to be undertaken must be taken into account for conditional reliability calculations. The system's conditional reliability function is given by:

$$R_S(T \mid t) = \frac{R_S(T+t)}{R_S(T)}. \tag{6}$$

Eq (6) gives the reliability for a new mission of duration t, having already accumulated T hours of operation up to the start of this new mission, and the system is checked out to assure that it will start the next mission successfully.

4.6 *Reliability Allocation and Optimization*

Consider a system consisting of n components. A reliability goal is sought for this system. The objective is to allocate reliability to all or some of the components of that system, in order to meet that goal with a minimum cost. The problem can be formulated as a nonlinear programming problem as follows [Ref. 1]:

$$P: \quad \min \quad C = \sum_{i=1}^{n} c_i$$
$$s.t. \quad R_s \geq R_G \tag{7}$$
$$R_{i,\min} \leq R_i \leq R_{i,\max}, \quad i = 1, 2, ..., n.$$

This formulation is designed to achieve a minimum total system cost, subject to R_G, a lower limit on the system reliability.

4.7 *Reliability Importance*

Importance measures provide a method of identifying the relative importance of each component in a system with respect to the overall reliability of the system. The reliability importance, *I*, of component *i* in a system of *n* components is given by [Ref. 2]:

$$I_{R,i}(t) = \frac{\partial R_S(t)}{\partial R_i(t)}. \tag{8}$$

The value of the reliability importance given by eq (8) depends both on the reliability of a component as well as on its corresponding position in the system.

5. EXAMPLE

The following example illustrates the steps and procedure for analyzing a complex system. A software package (BlockSim^{TM}) with the capability of solving for the analytical form of the system reliability is used. This software package includes all the equations described in Section 4. These equations are solved numerically based on the reliability equation.

5.1 *Application to a Complex System*

Consider the system shown in Figure 1 with the component information provided in Table 1. Obtaining the analytical solution of such a system can be rather time-consuming if it is done manually. Even a numerical determination of the system's reliability equation can be a challenge. Recent advances in both hardware and software, however, make it possible for such systems to be analyzed instantaneously, and without resorting to simulation methods.

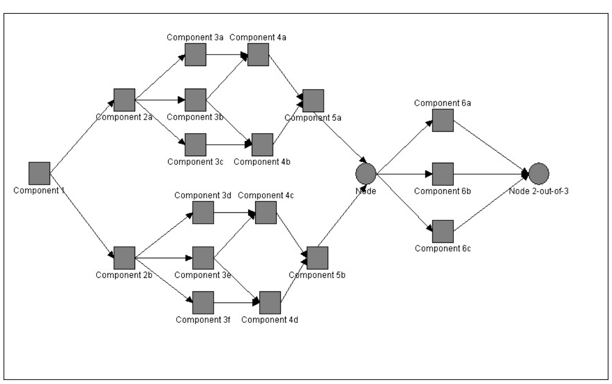

Figure 1: Reliability Block Diagram

Table 1: Example Component Data

	Distribution	**Parameters**
Component 1	Weibull	b =1.45 h = 15,000
Components 2a, 2b	Weibull	b =4.5 h = 11,799
Components 3a, 3b, 3c, 3d, 3e, 3f	Weibull	b =0.387 h = 2,466
Component 4a, 4b, 4c, 4d	Lognormal	m =7.922 s = 0.679
Component 5a, 5b	Lognormal	m =8.664 s = 0.272
Component 6a, 6b, 6c	Weibull	b = 0.5 h = 3,000

The equation for the system reliability obtained from BlockSim^{TM} is shown in Figure 2. Due to the length of the equation, only a part of it is shown in Figure 2. The total processing time for obtaining the reliability equation was approximately 4 seconds.

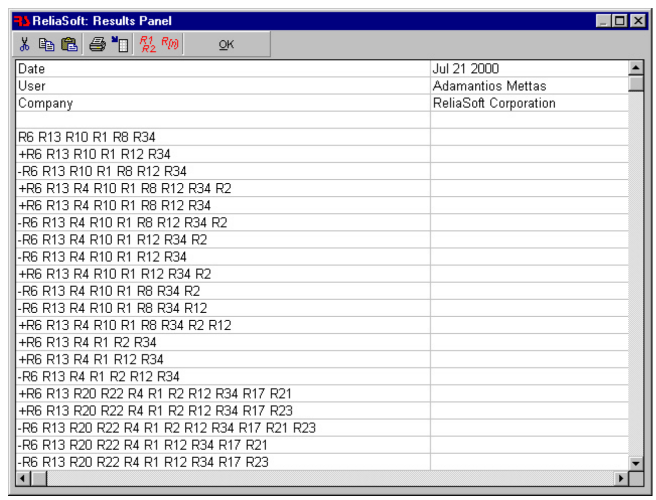

Figure 2: Example Reliability Equation

Using the reliability equation of the system, the reliability of any mission time can be calculated. In addition, a reliability vs. time plot can be generated, as shown in Figure 3.

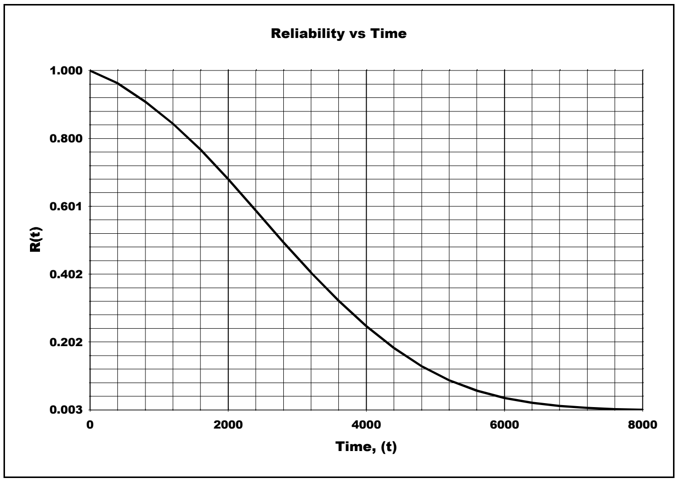

Figure 3: Reliability Plot

Using eq (2), the system's *pdf* can be obtained. The *pdf* can then be plotted as shown in Figure 4. Although the *pdf* plot is not as popular (for engineering purposes) as the Reliability and Failure Rate plots, it indicates the likelihood of failure at any time *t.*

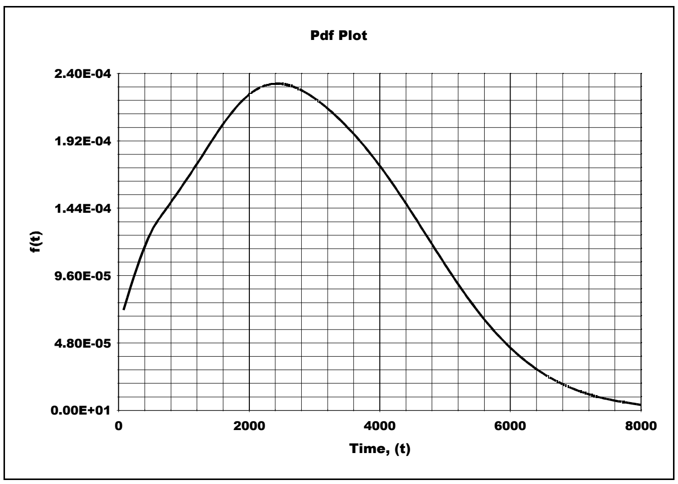

Figure 4: *pdf* Plot

A very popular and important metric in system analysis is the failure rate and the corresponding failure rate plot. Failure rate plots are very useful since they indicate the failure pattern of the system. A failure rate plot is shown in Figure 5, where an increasing failure rate with time pattern (wear-out) is observed for this particular system.

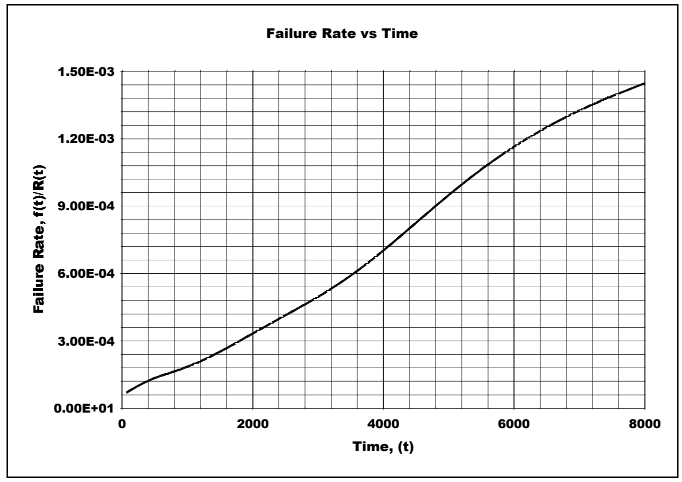

Figure 5: System Failure Rate Plot

A very important part of system analysis is in identifying the weak components and the components that have the greatest effect on the system reliability. From Figure 6 it can be seen that at 1,000 hrs Component 1 has the greatest reliability importance in the system. From a reliability point of view, it may make more sense to concentrate improvement efforts on Component 1, in order to increase the reliability of the system.

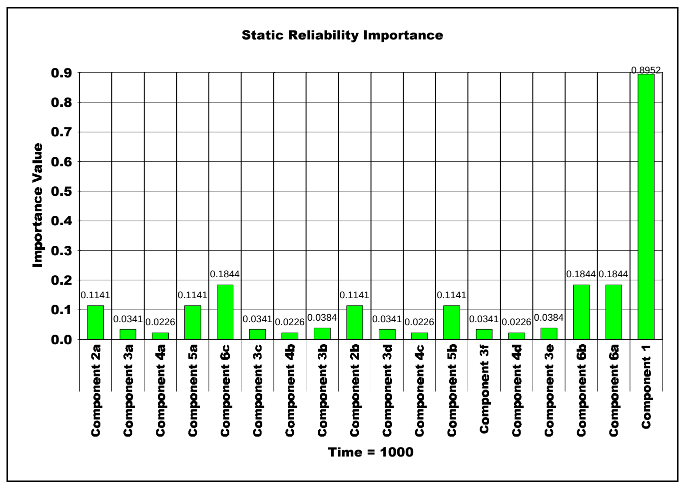

Figure 6: Component Reliability Importance

Note that the reliability importance of a component is time-dependent, as shown in Figure 7 (assuming its probability of failure is time-dependent). For this reason, the reliability importance must be obtained at the point of interest. For example, this point might be the time at which warranty is to be offered.

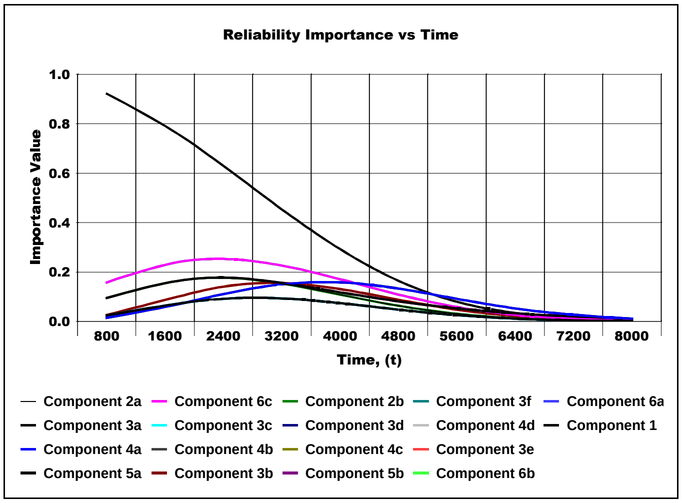

Figure 7: Component Reliability Importance vs. Time

Continuing the analysis of the system, consider the case where a reliability specification is to be met. The specifications for this system dictate that its reliability at 1,000 hrs is to be at least 95%. Calculating the system reliability at 1,000 hrs using the analytical solution, it is determined that the specification has not been achieved ($R_S(1000)$=87.77%). From Figure 6 it can be seen that Components 1, 6a, 6b, and 6c have the highest reliability importance. It is therefore concluded that efforts of improvement should concentrate on these four components. Using the optimization method described in Section 4.6, the reliability goal for these two components will be calculated in order to reach the system reliability goal of 95%. The optimization parameters [Ref. 1] for these two components are given in Table 2.

Table 2: Optimization parameters

	Maximum Achievable Reliability*	**Feasibility***
Component 1	0.9999	Hard
Components 6a, 6b, 6c	0.9999	Easy
*For more information on these two parameters see Ref. 1.		

The results of the optimization are given in Table 3. Note that such an optimization technique can also be applied when using simulation. However, this would mean that at every iteration of the optimization algorithm a significant number of simulations must be performed.

Table 3: Optimization Results

	***R*(1000)**	**R_{goal}(1000)**
Components 6a, 6b, 6c	0.5614	0.7150
Component 1	0.9805	0.9947
System Reliability	0.8777	0.9500

Given the allocation suggested in Table 3, the reliability goal of the components can be achieved either by fault tolerance or fault avoidance. Fault tolerance would require increasing the redundancy in the system, where fault avoidance would require redesigning Components 1, 6a, 6b, and 6c, or choosing a different supplier. Note that in the second case the failure distribution properties of the components can be changed. Since the structure of the system will not change, the reliability of the improved design can be calculated without the need of obtaining the system's reliability equation again. It will therefore be a straight substitution of the new distributions into the existing system reliability equation.

6. COMPARISON TO SIMULATION METHOD

In Section 5, a sample system analysis was performed and various analytical results were provided. To bring things into perspective, these results will be compared with results obtained using simulation. The comparison between the two methods is performed both on the accuracy of the results as well as on the processing time. The number of simulations used for the comparison was 20,000.

Table 4: Comparison of Analytical and Simulation methods

	Reliability *R*(1000)	Failure Rate l(1000)	*MTBF*	B10 Life	Importance	Optimization
Analytical* (Calculation Time)**	0.8777 (< 1 sec)	1.833E-4 (< 1 sec)	2913 (≈ 14 sec)	857.1 (≈ 3 sec)	Yes	Yes
Simulation (Simulation Time)**	0.8803 (≈ 28 sec)	2.157E-4 (≈ 28 sec)	2931 (≈ 28 sec)	N/A	N/A	N/A
*Does not include processing time for obtaining the reliability equation of the system.						
**Based on a P450, 96MB system.						

In Table 4, the processing times for the analytical calculations are provided, excluding the processing time for obtaining the reliability equation of the system. This value is approximately 4 seconds for this particular system. However, the system equation is obtained once, and unless there is a change in the structure of the system, it does not need to be recalculated. Once the system reliability equation is obtained, all further calculations can be performed almost simultaneously. For example, the reliability of the system at 1,500 hr. can be determined in less than 1 second, as can any other reliability value. On the other hand, in order to solve for the reliability at 1,500 hr using simulation, another 20,000 simulations will be required. Note that decreasing the number of simulations can reduce the processing time for the simulation. This however means that the precision on the estimates is reduced as well.

Clearly, analytical solutions offer great power and speed in system analysis, and that is also the main reason why they are becoming more and more popular over the simulation methods. The biggest disadvantage of the analytical method is that there are cases where formulations can become very complicated. The more complicated a system is, the larger and more difficult it will be to analytically formulate an expression for the system's reliability. For particularly detailed systems, this process can be quite time-consuming, even with the use of computers. Furthermore, when the maintainability of the system or some of its components must be taken into consideration, an analytical solution may be impossible to compute. In these situations, the use of simulation methods may be more advantageous than attempting to develop a solution analytically.

Future development of system analysis software should concentrate on obtaining analytical solutions for complex repairable systems, and in particular on repairable systems with non-exponential failure and repair rate components.

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