Warranty Prediction for Parts with Design Changes

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SUMMARY & CONCLUSIONS

Warranty prediction is a very important task in reliability engineering. It needs to estimate the expected number of failures in any given time period during the length of the warranty contract. Several commercial software packages have been already implemented and used in the industry, including Minitab and Weibull++. The time to failure is usually selected to be a Weibull distribution and no technology improvement in the manufacturing process or in the product design is assumed. This paper introduces a new mathematical model which provides the requested predictions under much more general conditions.

It is very common that the design and the manufacturing process of an item will change to fix issues discovered in the field. These changes will result in the change of the failure behavior which is often modeled by a time to failure distribution such as Weibull. In our model we consider a manufacturing plant producing identical items in given numbers during each time period. They are subject to possible failures in any later time period after they are produced and the replacements also can fail later as newly produced items.

It is assumed that at a given later time, the technology changes so the time to failure distribution also changes, and all items which are replaced or produced from this time period will follow the new time to failure distribution. In order to plan appropriate inventory strategy it is necessary to predict the expected total number of failures in every time period during the considered warranty time interval. In computing the total number of failures the cumulative effect of the failures of new items as well as those of their possible replacements have to be considered and taken into account.

A mathematical model is first introduced, where, for the sake of simplicity, it is assumed that after introducing the new technology the produced or replaced items will no longer fail. The general case can be, however, considered and solved in a similar way.

In addition to an analytic solution methodology a simulation study is presented. The Weibull distribution is used in the numerical example, however, it can be replaced with any other distribution type.

It is demonstrated that the usual prediction method can be successfully extended into cases when the improvement of the production technology changes the distribution of the time to failure, and therefore the probabilistic properties of all items produced or replaced after this change are also changed. The expectation of the cumulative number of failures in each time period provides important help in finding the most appropriate inventory strategies leading to significant savings in inventory cost as well as in the cost of delayed services.

1 INTRODUCTION

Finding appropriate maintenance and replacement policies is one of most important issues in reliability engineering discussed in a wide range of literature; for example [1], [2], [3], [4], [5], [6] among others. In order to make optimal, or at least, acceptable maintenance planning decisions, the expected number of failures should be estimated. In many cases, there are simple mathematical models for this estimation. For example, if minimal repair is made, then the expected number of failures in any time interval $[t_1, t_2)$ is

$$M(t_1, t_2) = \int_{t_1}^{t_2} \rho(t)dt \tag{1}$$

where $\rho(t)$ is the failure rate. If failure replacement is performed at each failure, then $M(0, t)$ satisfies the integral equation.

$$M(0, t) = F(t) + \int_0^t M(0, t - s)f(s)ds \tag{2}$$

where $F(t)$ and $f(t)$ are the CDF and pdf of time to failure, respectively.

In many manufacturing facilities, the problem might become much more complicated, since technology changes rapidly and therefore the time to failure distribution also changes.

In this paper we make the first attempt to deal with this issue with special assumptions, which will then help the researchers to address more complex situations such as warranty prediction with multiple design changes.

2 PROBLEM DESCRIPTION

Consider a manufacturing facility producing identical items with a known distribution of time to failure $F(t)$. In the case of a failure, the item is replaced by a new one. Discrete time scale is considered, $t = 0,1,2,\ldots$ and the time intervals are indexed as 0 for $[0,1)$, 1 for $[1,2)$, and so on.

Assume that N_k items are produced in interval k, and for the sake of simplicity it is assumed that they are produced at the beginning of this time interval, and if a failure occurs during an interval, then it is replaced at the end of that interval. From time T, a new technology will be used to fix a discovered issue. Any item produced or replaced at time T or after will have no breakdowns afterwards, if the fix is perfected. If the fix is not perfect, a new failure time distribution should be used for items produced after time T. Time T sometimes is called the clean point.

The problem, then, is to find the expected number of failures in any interval $[t, t+1)$, so that the future maintenance schedules and/or inventory can be planned in advance, and spare parts or completed items can be ordered optimally.

3 MATHEMATICAL MODEL

Consider first an item which is produced at the beginning of time interval $k = [k, k+1)$ and consider a later interval, $t = [t, t+1)$. For the sake of simplicity we assume that at most one failure of this item occurs at any time interval, the more general case where many items can be replaced in time interval k can be handled by including the $M(t, t+1)$ values as multipliers. Let $P(t,k)$ denote the probability that the item produced at the beginning of interval k will have a failure in interval t. Clearly, $P(t,k) = 0$ for $t < k$.

We will also use the simplifying notation $F_{ji} = F(j) - F(i)$ for $i \leq j$.

Assume first that $k \leq t \leq T$. Obviously

$$P(k,k) = F(1) - F(0) = F_{1,0} \tag{3}$$

similarly,

$$P(k+1,k) = F_{2,1} + P(k,k)F_{1,0} \tag{4}$$

depending on the condition, if the first failure occurs in interval $k+1$ or in interval k. In the same way we can see that

$$P(k+2,k) = F_{3,2} + P(k,k)F_{2,1} + P(k+1,k)F_{1,0} \tag{5}$$

The first term refers to the case when first failure occurs in interval $k+2$, the second term when it occurs in interval k and no more failures until interval $k+2$ and the last term shows the case when failure occurs in the previous interval $k+1$. Using the same idea we can show that in general

$$P(t,k) = F_{t-k+1,t-k} + \sum_{l=k}^{t-2} P(l,k)F_{t-l,t-l-1} + P(t-1,k)F_{1,0} \tag{6}$$

It is also an interesting question to find the probability that failure occurs in interval $t < T$ and no more failure will occur afterwards. It is very easy to see that this probability equals

$$B(t,k) = P(t,k)\left(1 - F_{T-t-1,0}\right) \tag{7}$$

where the second factor is the probability that no failure occurs between time $t+1$ and T.

Assume next that $t > T$, then

$$P(t,k) = F_{t-k+1,t-k} + \sum_{l=k}^{T-1} P(l,k)F_{t-l,t-l-1} \tag{8}$$

since the last failure before time T can be in intervals $k, k+1, \ldots, T-1$.

And finally, if $k \geq T$, then the item is produced with new technology, so no failure will occur if the fix is perfect, implying that $P(t,k) = 0$. Otherwise, a new failure time distribution should be used to calculate $P(t,k)$ and F_{ji} for items produced after T.

In the previous derivation we computed the probabilities that an item produced at the beginning of interval k will break down in intervals $k, k+1, k+2, \ldots, T-1, T, \ldots$ The manufacturing facility produces N_k items at the beginning of each time period k. They will never break down if $k \geqq T$, and the probability of break down is computed above for each of these items in all future intervals $k, k+1, \ldots$ Therefore the expected number of failures in any interval $t = [t, t+1)$ is obtained by the simple formula for expectation:

$$M(t,t+1) = \sum_{k=0}^{T-1} N_k P(t,k) \tag{9}$$

Notice that if $k \geqq T$, then no failures occur due to the new technology. In general

$$M(t,t+n) = \sum_{l=t}^{t+n-1} M(l,l+1) \tag{10}$$

gives the expected number of failures in any interval.

4 CASE STUDY

The analytical methodology found earlier in this paper can be solved using event based simulation. The simulation results can be used to validate the proposed analytical method in this paper.

For the simulations we considered two cases, one in which the new design can fail, and one in which the new design cannot fail.

All events generated during the simulations were based on the following models and assumptions:

1. 100 items are produced and put into service during each time interval. In this case each interval represents one month.
2. All replacements are performed at the end of the interval in which the failure occurs.
3. The initial failure distribution is assumed to be a 2-parameter Weibull with a $\beta = 1.5$ and $\eta = 1460$ hours.
4. The new technology is available after the 6^{th} interval, where the parameters of the failure distribution change to $\beta = 1.5$ and $\eta = 2920$ hours for the case when the new design can fail.

In both cases, we were interested in two statistics. The first statistic is the expected number of failures during each time interval. Figure 1 shows the expected number of failures when the new design can still fail, while Figure 2 shows the expected number of failures when the new design cannot fail.

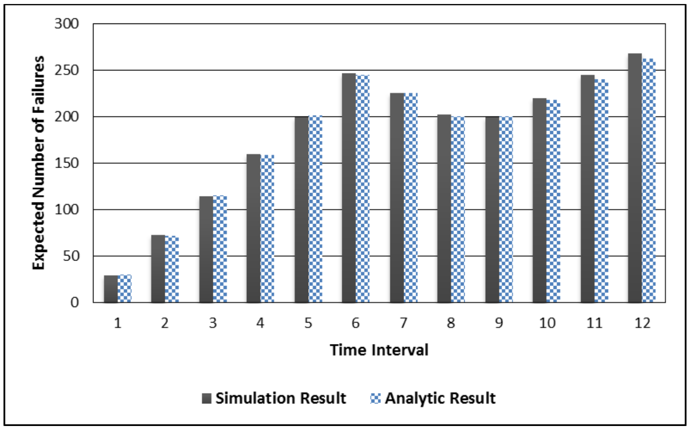

Figure 1. *Expected number of failures during each interval when the new design can fail.*

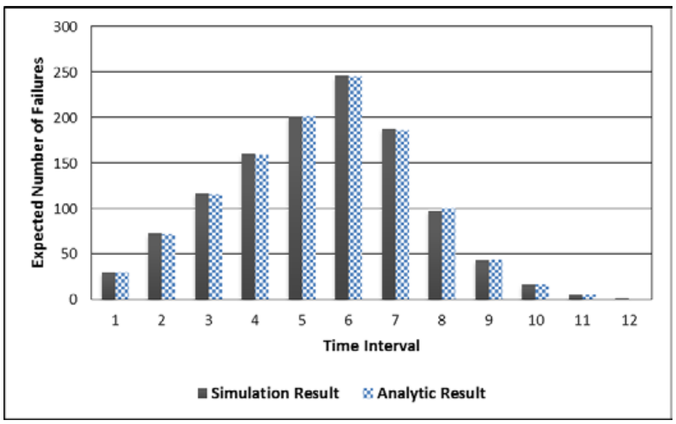

Figure 2. *Expected number of failures during each interval when the new design cannot fail.*

The second statistic is the expected number of replacements for items produced in a given interval. Figures 3 and 4 show the expected number of replacements for items originally produced during a given time interval, where Figure 3 is for the case when the new design can fail, and Figure 4 is for the case when the new design cannot fail.

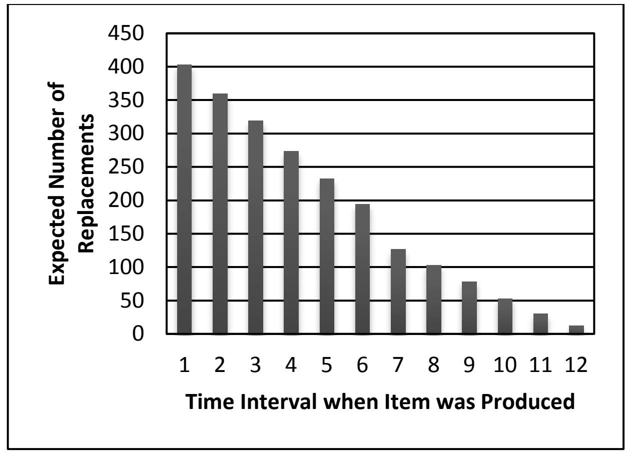

Figure 3. *Expected number of replacements for items produced in the given interval when the new design can fail.*

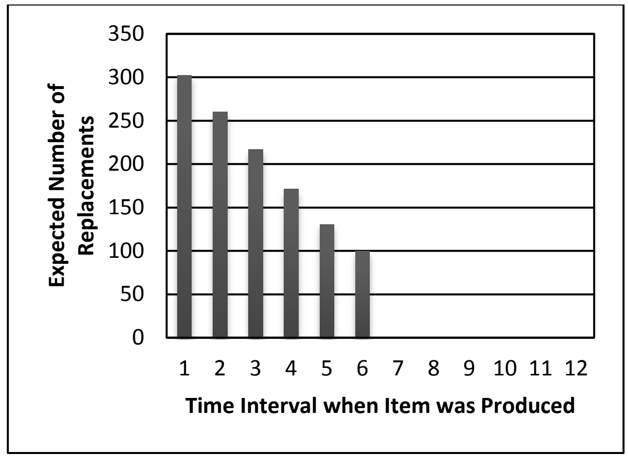

Figure 4. *Expected number of replacements for items produced in the given interval when the new design cannot fail.*

REFERENCES

1. T. Dohi, N. Kaio, S. Osaki, "Preventive maintenance models: Replacement, repair, ordering, and inspection." *Handbook of Reliability Engineering Pham, H. (ed.).* Springer, London, 2003.
2. A. Jardine, A. Tsang, *Maintenance, Replacement, Reliability: Theory and Applications*, Boca Raton, CRC Press, 2013.
3. T. Nakagawa, *Advanced Reliability Models and Maintenance Policies*, London, Springer, 2008.
4. T. Nakagawa, *Random Maintenance Policies*, London, Springer, 2014.
5. T. Nakagawa, X. Zhao, *Maintenance Overtime Policies in Reliability Theory: Models with Random Working Cycles,* London, Springer, 2015.
6. S. Osaki, *Stochastic Models in Reliability and Maintenance,* London, Springer, 2002.

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Miklós Szidarovszky is a Technologist at HBM Prenscia. He is involved in the development of various ReliaSoft software products, the delivery of training seminars, and consulting projects. His areas of interest include Life Data Analysis, Accelerated Testing, System Reliability, Probabilistic Event and Risk Analysis, and Risk Based Inspection. Mr. Szidarovszky holds a B.S. and an M.S. in Chemical Engineering from the University of Arizona. He is a Certified Reliability Professional (CRP) and a Certified Scrum Master (CSM).

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Ferenc Szidarovszky is a senior researcher with the Ridgetop Group Inc. working on theoretical issues concerning reliability, robustness, optimal maintenance and replacement policies and prognostics in complex systems. In addition to these industrial applications he also performs research in systems theory, especially in dynamic economic systems. He has earned two PhD degrees, one in Applied Mathematics and another in Economics in Hungary. The Hungarian Academy of Sciences has awarded him with two higher degrees, Candidate in Mathematics and Doctor of Engineering Sciences. After graduating in Hungary, he became a professor of the Eotvos University of Sciences, Budapest and later became an acting department head at the Budapest University of Horticulture and Food Industry. In 1987 he moved to the US and from 1988 he was a professor of the Systems and Industrial Engineering Department at the University of Arizona. From September 2011 he was a Senior Researcher at ReliaSoft Corporation, and as of July 2015 he is with Ridgetop Group. As a Hungarian professor he was an investigator in the first US-Hungarian joint research project jointly supported by the NSF and Hungarian Academy of Sciences on Decision Models in Water Resources Management. He has performed research in very broad fields of applied mathematics including numerical analysis, optimization, multi-objective programming, game theory, dynamic systems, applied probability and their applications in industrial processes, environment, water resources management, economics, computer network security among others. He is the author of 21 books and over 350 articles in international journals in addition to numerous conference presentations and short courses taught in many countries. He currently also has a professor position with the Applied Mathematics Department of the University of Pecs, Hungary, where his duty is a one-week long intensive Game Theory Course every year.

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