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Reliability Growth Planning Curves Based on Multi-Phase Projections

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Key Words: Growth Planning, Projections, Extended Model

SUMMARY & CONCLUSIONS

The planned reliability growth curve gives the interim milestone reliability targets in which the system is judged as the reliability growth program progresses across single or multiple test phases. At the end of each test phase the system reliability is assessed and compared to the corresponding value on the planned reliability growth curve. If the assessed reliability is on or above the milestone value the reliability program continues. If the assessed reliability is below the milestone value changes in the reliability program may be necessary. It is therefore important that the planned reliability growth curve be realistic and based on input parameters and assumptions that adequately reflect the configuration and characteristics of the system.

In many cases some of actual parameter values during testing are different than the values inputted into the planning model. These differences may or may not affect meeting the reliability goals set by the initial planning curve. However, if there is a negative impact regarding meeting the objectives in the next and future test phases then addressing this by management and engineering in a timely manner is very important. The focus of this paper is to provide a methodology to progressively update the planned reliability growth curve across all future test phases based on actual test data and give projections for the expected reliability over multiple future test phases. This methodology is currently being used to provide confidence that a program is on track or to identify risks.

1 INTRODUCTION

The attainment of reliability requirements for complex, state of the art systems has historically been a challenge for many decades, which prompted Selby and Miller of General Electric in 1971 to propose in their paper Reliability Planning and Management (RPM), Ref. 1, that Reliability Growth Planning curves be utilized. The reliability growth planning curve methodology in their paper is based on a 1962 paper by J. T. Duane, of GE, Ref. 2, in which Duane postulated a pattern for reliability growth based on development data at GE. To construct the Selby and Miller RPM Reliability Growth Planning Curve several parameters are put into the model including the expected starting MTBF and the growth rate. The growth rate concept is from Duane's paper and is a measure of how fast the reliability growth will occur. The fundamental application of the RPM concept is to construct the Planned Reliability Growth curve with the stated growth rate as a function of test time and find the test length T such

that the curve at test time T meets the requirement or reliability goal. The necessary test time T to meet the reliability goal is a dependent variable.

Crow modified the Duane RPM planning model in Mil Hdbk 189, 1981, Ref. 3, by a re-parameterization so that the initial average MTBF over the initial test phase is an input parameter. This model maintained (1) the concept of growth rate and (2) the objective of determining the test time T necessary to meet the requirement or goal.

Another Reliability Growth Planning model that includes the concept of growth rate and the objective of determining the test time T necessary to meet the requirement or goal is the Extended Planning model developed by Crow, Ref. 4. For this model the parameter driving the growth rate is the distinct failure mode discovery parameter β_D . In Ref. 4 it is shown that $\alpha = 1 - \beta_D$ is analogous to the Duane growth rate, and, based on historical data a typical historical value for β_D is about 0.70 (see Ref. 4).

The Extended Planning model has parameters that are more extensive than the RPM model and the Crow/Duane 1981 Mil Hdbk 189 model. These parameters include the Type A mode (failure modes that are not corrected) failure rate and Type B mode (failure modes are corrected) failure rate, growth potential, fix effectiveness factors, and the discovery function which was developed by Crow in Ref. 5. In Ref. 6 Crow showed that the discovery function, the rate that new Type B failure modes requiring corrective action are being seen as the testing is progressing, must follow the Crow (AMSAA) model if the reliability growth is assumed to follow the Duane and Crow (AMSAA) reliability growth pattern. For the discovery function only the first occurrences of new failure modes are included for reliability growth tracking. With this model the necessary test time T to meet the reliability goal is a dependent variable.

Milena Krasich, 2014, in Ref. 7 asserts that the concept of using Type A modes, Type B modes, and a Type B mode discovery function based on the Crow (AMSAA) model has not been historically done and proposed a model. This assertion is incorrect. Type A modes, Type B modes, together with the Crow (AMSAA) model Type B mode discovery function are the basis of the 2011 Crow Extended Planning model in Ref. 4. On closer examination of Ref. 7, equations 3 and 4, the model proposed by Krasich in Ref. 7 is equal to Constant failure rate plus the Crow (AMSAA) model or

$$r_{Planning}(t) = \lambda_{Constant} + \lambda\beta t^{(\beta-1)} \quad (1)$$

This equation is exactly equal to the Crow Extended Reliability Growth Planning model Ref. 4, equation 21, and in this paper, equations 6 and 10, below, in which the average fix effectiveness factor parameter d is taken as $d = 1$ by Krasich. That is, the reliability growth planning model proposed by Krasich was already developed in a more general form by L. H. Crow in 2011 and has been widely used since.

The Selby and Miller model, the 1981 Crow/ Duane based Mil Hdbk 189 model, and the Extended planning model all have the characteristic of growth rate and the risk assessment capability of estimating the amount of reliability growth test time that is needed to attain the requirement or goal. There are other planning models that do not have these characteristics.

Over the past few years the Department of Defense has promoted, and often required another reliability growth planning model, the PM2 model in Ref. 8. This model has different characteristics and objectives than the Duane based Mil Hdbk 189 planning model and the Extended Planning model. The main characteristic of the PM2 is that for any given reliability growth test time T the PM2 model MTBF curve always goes through the MTBF goal at time T . For example, if the PM2 model gives a planning curve that will achieve a fixed MTBF goal at a user input time of 5000 hours, it will also give a planning curve that meets the same MTBF requirement for user input time of 500 hours for all other inputs exactly the same. For the PM2 model there is no user historic growth rate input for planning, and the PM2 model cannot be used to estimate the required test time to meet the goal based on historical experienced growth rates. For the PM2 model the test time T to meet the reliability goal on the growth curve is not derived but is a user input. It cannot be assumed that the PM2 planning curve is realistic based on historical experiences and that time T reflects sufficient test time.

For the Crow/Duane based Mil Hdbk 189 (1981) Planning model and the Extended Planning model the necessary test time T to attain the requirement is estimated utilizing the model input parameters which should be based on historical data and the other system, engineering and management characteristics. However, there is a risk that at least one of the input parameters may not be consistent with the actual test results which could indicate insufficient test time or other issues affecting meeting the requirement. In additional, for the PM2 model there is the risk that the user allocation input T may be too low which forces the equivalent growth rate to be too high so that the requirement cannot be attained during the $(0, T)$ reliability growth test.

A typical reliability assessment at the end of a test phase is to compare the demonstrated or projected reliability to the corresponding value on the planned reliability growth curve. If this assessed reliability is equal to or greater than the value on the curve then the program is generally assumed to be on track to meet the requirement. This conclusion may not, however be correct. The objectives of this paper are to present a methodology to use early test data to assess the parameters of the planning model, estimate these parameters, develop an updated planning model, and provide information regarding

the risks associated with meeting the reliability interim goals in future test phases and meeting the reliability requirement at time T , the end of reliability growth testing. The methodology proposed in this paper utilizes the Extended Continuous Evaluation projection model Ref. 6 which estimates the parameters of interest in the Extended Planning model Ref.4. This model uses test data over one or multiple test phases for estimation and projections,

2 RELIABILITY GROWTH PLANNING MODELS

2.1 Duane Postulate

In 1962 J. T. Duane of General Electric published a paper, Ref.1, in which he plotted data from several systems undergoing reliability growth testing at General Electric. For each system Duane plotted the cumulative or average failure rate (intensity) at various time points during the testing. If $N(t)$ is the cumulative number of failures up to time t during the testing Duane calculated $N(t)/t$ the cumulative failure rate $C(t)$.

Duane then plotted $\ln(C(t))$ versus $\ln(t)$. As observed by Duane the plots were close to being linear when plotted on \ln - \ln scale. Duane called the negative of the slope of the straight lines the "growth rate."

In 1972 Crow, see Ref. 2 and Ref. 3 addressed the issue of establishing a statistical framework for reliability growth data that has the empirical properties noted in the Duane Postulate. Crow modelled the reliability growth failures times as a non-homogeneous Poisson Process (NHPP) with failure intensity given by

$$r(t) = \lambda \beta t^{\beta-1}, \quad (2)$$

thus allowing for statistical procedures based on this process for reliability growth analyses. The growth rate is given by

$$\alpha = 1 - \beta. \quad (3)$$

In Mil HDBK 189, Reliability Growth Management, L. H. Crow parameterized the Duane prostate for reliability growth planning as

$$M(t) = M_1 \left(\frac{t}{t_1} \right)^\alpha (1 - \alpha)^{-1} \quad (4)$$

where M_1 is the average MTBF over the first test phase $(0, t_1)$.

2.2 Extended Planning Model

An Idealized Reliability Growth Planning Curve is the reliability at time t if all Type B modes that have been seen at time t are fixed. This is what the 1982, Ref. 5, projection Model estimates. The application of the 1982 Projection model as an Idealized Growth Curve concept was first proposed by Crow in 1984. Also, in 1989 Gibson and Crow, Ref. 8, showed that under reasonable conditions we can use an average Fix Effectiveness Factor d and write the 1982 projection model as

$$\lambda_{\text{projection}} = \lambda_A + (1-d)\lambda_B + d \cdot \lambda_D \beta_D t^{\beta_D-1} \quad (5)$$

The 1982 Projection model, Ref. 5, is a special case of the

Extended Continuous Evaluation Projection Model, Ref. 6. For projections the parameters in the projection model are estimated and the effectiveness factor is an input based on historical experiences. For planning, key parameters are inputs. This is the Extended Reliability Growth Planning Model, Ref. 4, defined in terms of failure intensity as

$$\lambda(t)_{\text{Extended-Planning}} = \lambda_A + (1-d)\lambda_B + d \cdot \lambda_D \beta_D t^{\beta_D-1} \quad (6)$$

and in terms of MTBF as

$$M(t)_{\text{Extended-Planning}} = \left[\lambda_A + (1-d)\lambda_B + d \cdot \lambda_D \beta_D t^{\beta_D-1} \right]^{-1}. \quad (7)$$

The Extended Planning Model has input parameters which include (1) the total test time T (2) the requirement MTBF (3) the management strategy MS (4) the Growth Potential Design Margin, GPDM (5) the average Fix Effectiveness Factor (FEF) d, and (6) the discovery parameter β_D in the h(t) function. With these inputs the output is the final MTBF at time T with the corresponding planned growth curve and test phase targets. The total test time T that is necessary to meet the requirement can be evaluated with this model to assess the risk associated with the plan based on historical experiences and data.

2.3 PM2 Planning Model

As noted in Ref. 8 the input parameters for the PM2 Planning model include (1) The total test time T (2) the requirement MTBF (3) the management strategy MS (4) the average FEF d, and (5) the initial MTBF. The PM2 model has a discovery function of the form (see Ref. 8)

$$h_{PM2}(t) = \frac{\lambda_B}{(1 + \beta_{PM2} \cdot t)} \quad (8)$$

with parameter β_{PM2} . If the goal MTBF is given, then the PM2 model methodology solves for this parameter such that the required MTBF is met at time T. The parameter β_{PM2} is solved and is not a user input. The output is a planned growth curve that always goes through the goal MTBF at test time T.

3 GROWTH POTENTIAL AND DISCOVERY FUNCTION

As discussed in Ref. 5,

$$\lambda_{GP} = \lambda_A + (1-d)\lambda_B \quad (9)$$

is the Growth Potential failure intensity for an average Fix Effectiveness Factor d. Therefore, the Extended Planning model can be expressed as

$$\lambda(t)_{\text{Extended-Planning}} = \lambda_{GP} + d \cdot \lambda_D \beta_D t^{\beta_D-1} \quad (10)$$

where λ_{GP} denotes the Growth Potential failure intensity and

$$h(t) = \lambda_D \beta_D t^{\beta_D-1} \quad (11)$$

is the Crow(AMSAA) model rate of discovering new Type B modes at time t. The Crow (AMSAA) model is based on the nonhomogeneous Poisson process, and has the same growth pattern as the Duane postulate discussed above.

A very important note is that the rate in which new Type B failure modes are being discovered at time t is exactly equal to the total B mode failure rate that is unseen at time t. This

means that λ_B is the total unseen B mode failure intensity at time t=0, and is also the instantaneous rate in which we are discovering new B failure modes when testing starts at time t = 0. In addition, as B modes are corrected by time t, and corrective actions are effective, the seen B modes after the fix actually become, at time t, Type A modes. The Type A failure intensity gets larger and the Type B failure intensity gets smaller and equals at time t h(t) given by equation (11). This implies that the Growth Potential can be reassessed utilizing an estimate of h(t) based on data during the test. This is what the Extended Continuous Evaluation model does and is based on the actual Crow (AMSAA) model estimate of the Demonstrated MTBF and FEF for current and future corrective actions.

With the Extended Continuous Evaluation model no FEFs are assumed for corrective actions noted as fixed prior to time t, as assessment point. The actual reliability growth due to prior corrective action is estimated from the data, and the impact of corrective actions incorporated at time t or later utilizes data plus FEFs. Among the parameters estimated at time t are the Growth Potential, and the parameters λ_D and β_D in the h(t) discovery function.

The parameter β_D must be less than one for reliability growth. If $\beta_D = 1$ then there can be no reliability growth regardless of the value of the Growth Potential MTBF. This is because a value of β_D equal or close to one implies that the system has lots of problem Type B failure modes each having very small failure rates, so that correcting these will have little or no impact on improving reliability. At the other extreme is a β_D that is very small. This implies that there are a very few failure modes in the system that have large failure rates, so that correcting these few problems will have a dramatic impact on reliability approaching the Growth Potential very rapidly. The first situation is not desirable for reliability growth and the second situation is unlikely for a complex system. Historical data and experience indicate that the common situation is a β_D in the range near 0.7.

4 THE EXTENDED CONTINUOUS EVALUATION RELIABILITY GROWTH PROJECTION MODEL

With the Extended Continuous Evaluation model reliability analyses may be conducted multiple times during the test program and, in particular, at the end of each test phase. A test phase usually ends at a planned stopping point in the test program. There are generally delayed corrected actions incorporated into the system before the next test phase starts. An additional general feature of a reliability growth test phase is that there are usually interim requirements or goals to be achieved based on the planning curve. Therefore, there are generally assessments of the current reliability and a projection of the reliability expected in the next test phase.

Each time we make an assessment with the Extended Continuous Evaluation model we can calculate estimates of several key metrics which include: (1) Current Demonstrated MTBF (2) Nominal Growth Potential MTBF (3) Rate of

discovery.

5 RELIABILITY GROWTH PLANNING CURVES AND MULTI PHASE PROJECTIONS

Suppose a reliability growth program consists of K test phases at times T_1, T_2, \dots, T_K . At the end of phase 1 at T_1 corrective actions are incorporated into the system based on some or all of the Type B modes seen up to time T_1 . The Extended Continuous Evaluation model's nominal Growth Potential and the nominal Projection are estimated at time T_i are calculated based on the assumption that all seen Type B failure modes are corrected.

At time T_i the Extended Continuous Evaluation model nominal projection failure intensity λ_{NP} is estimated by

$$\hat{\lambda}_{NP} = \hat{\lambda}_{NGP} + d_N \cdot \hat{h}(T_i) \quad (12)$$

where $\hat{\lambda}_{NGP}$ is the estimated nominal projection failure intensity at time T_i , $\hat{\lambda}_{NGP}$ is the estimated Nominal Growth Potential failure intensity utilizing data over the interval $(0, T_i)$, d is the average of the currently assigned FEFs, and $\hat{h}(T_i)$ is the estimated discovery function evaluated at time T_i .

5.1 Multi-Phase Projections

The extended continuous evaluation model nominal MTBF projection at time T_i estimates the system MTBF if all type B failure modes seen up to time T_i have been incorporated. In this paper we expand the concept of a projection at the end of a single phase and extrapolate the projection to the ends of future test phases. In particular we rewrite equation 12 as

$$\hat{\lambda}_{NP}(T_i) = \hat{\lambda}_{NGP}(T_i) + d_N \cdot \hat{h}(T_i) \quad (13)$$

where $\hat{\lambda}_{NGP}(T_i)$ is the estimated Nominal Projection failure intensity at time T_i , $\hat{\lambda}_{NGP}(T_i)$ is the estimated Nominal Growth Potential failure intensity utilizing data over the interval $(0, T_i)$, d is the average of the currently assigned FEFs, and $\hat{h}(T_i)$ is the estimated discovery function evaluated at time T_i .

The Extended Continuous Evaluation model nominal MTBF projection at time T_i is

$$MTBF_{NP}(T_i) = \left[\hat{\lambda}_{NGP}(T_i) + d_N \cdot \hat{h}(T_i) \right]^{-1} \quad (14)$$

Suppose the reliability growth test has completed the i -th test phase. The Extended Continuous Evaluation model failure intensity Projection at the end of test phase i is given by equation 14. Given that the reliability growth test has completed the i -th test phase we now expand the Extended Continuous Evaluation model projections to the ends of future test phases.

The Extended Continuous Evaluation model MTBF projections to the ends of multiple future test phases based on data to time T_i is given by

$$\lambda_{NP}(T_{i+j} | T_i) = \lambda_{NGP}(T_i) + d_N \cdot h(T_{i+j} | T_i) \quad (15)$$

where $\lambda_{NP}(T_{i+j} | T_i)$ denotes the nominal projection failure intensity based on data only over the interval $(0, T_i)$, and

$$\hat{h}(T_{i+j} | T_i) = \hat{\lambda} \hat{\beta} T_{i+j}^{\hat{\beta}-1} \quad (16)$$

uses $\hat{\lambda}, \hat{\beta}$ estimates based only on data over the interval $(0, T_i)$. The Multi-Phase nominal projected MTBF is

$$MTBF_{NP}(T_{i+j} | T_i) = \lambda_{NGP}(T_i) + d_N \cdot h(T_{i+j} | T_i) \quad (17)$$

for $j \geq 0, i + j \leq K$.

5.2 Planned Growth Curves from Multi-Phase Projections

When developing a planned reliability growth curve there are various assumptions and inputs into a model. For example, for the Extended Planning model the growth potential is an input, either directly or indirectly, together with the average FEF d , and the discovery beta, β_D . The discovery lambda, λ_D is estimated by the model based on certain assumptions. However, once we have data the Extended Continuous Evaluation model estimates these parameters for testing over a single test phase and testing over multiple test phases. Therefore, these estimates can be used to construct a planned reliability growth curve based on actual data. In particular, we can use the multi-phase projected MTBF equation in (15) to construction an idealized planned reliability growth curve given by

$$\hat{M}_{ExtendedPlanning}(t) = \left[\hat{\lambda}_{NGP}(T_i) + d_N \cdot \hat{h}(T_i) \right]^{-1} \quad (18)$$

for $t \geq 0$. Of course, from a management point of view we will, in general be most interested in the updated planning curve for t over the range (T_i, T_K) .

6 NUMERICAL EXAMPLES

In this section we will illustrate the methodologies presented in this paper by numerical examples. The example based on the Extended Continuous Evaluation model utilizes the example given in Section 4 of Ref. 6. The reader is referred to Ref. 6 for additional details.

6.1 Example 1

Suppose a complex network system being developed by Contractor XYZ has several subsystems. One of the subsystems will consist of a redundant k out of n node in the system. For the node to meet its reliability requirement the complex subsystem needs to have an MTBF of at least 11.5 hours. The contractor chooses to plan the reliability growth program using the Extended Planning model, Ref. 4. Based on the design, data analysis, similarly studies, and predictions, the inherent growth potential MTBF of the design is estimated to be 14.37, or a Growth Potential Design Margin (GPD) of 1.25. That is, the estimated potential MTBF of the design is 25% above the requirement.

The Management Strategy is the fraction of the initial

failure intensity that will be addressed by corrective actions. Because this is a complex, new technology system the contractor expects this to be high at 0.95. In the planning the contractor also uses Ref. 4 and assumes typical historical values of FEF of $d= 0.70$ and a beta discovery $\beta_D = 0.70$. This information is put into the Extended Planning Model, see Ref. 4, that calculates an estimate of the discovery lambda of

$$\lambda D = 0.378626 \quad (19)$$

and minimum initial MTBF over the first test phase of

$$MTBF = 4.88. \quad (20)$$

With the planning assumptions of $FEF = .7$, $\beta_D = 0.70$, we use eqs. (6) and (7) to calculate the Extended Planning model MTBF curve as

$$M(t)_{\text{Extended-Planning}} = \left[\frac{1}{14.37} + 0.70 \cdot (0.378626) \cdot 0.70 \cdot t^{(0.70-1)} \right]^{-1} \quad (21)$$

Using eq. (21) we calculate that about $T = 2700$ hours of reliability growth development testing is needed to attain the MTBF goal of 11.5. Contractor XYZ plans for three test phases ending at times $T_1 = 400, T_2 = 1500, T_3 = 2700$. This gives the planned reliability growth curve in Figure 1, with the initial Phase 1 MTBF =4.8, Phase 2 MTBF = 9.7, and Phase 3 MTBF = 11.0.

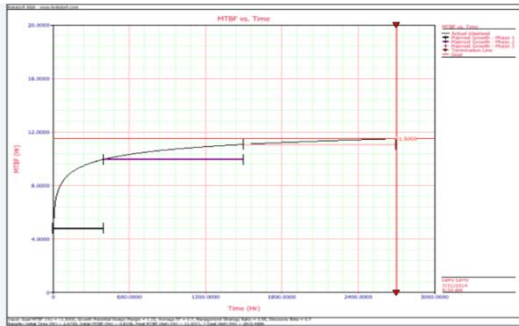


Figure 1. Extended Planning Model Growth Curve

Contractor XYZ then runs the first 400 hour phase of the reliability growth test with the results given in Tables 1. This is the same data in Tables 1 of Ref. 6. It is analyzed in Ref. 6 using the Extended Continuous Evaluation model. See Ref. 6 for more details. In Table 1 Type BC modes are modes corrected at the time of failure, for example safety related issues. Type BDC failure modes are modes seen during test, corrective actions are delayed to after the time of first occurrence, and the modes are corrected during the current test (0,400). Type BDD failure modes are modes seen during test, corrective actions are delayed to after the time of first occurrence, and the modes have not been corrected during the current test (0,400). When the first phase test is stopped at 400 hours all Type BDD modes can be corrected or some can be delayed and fixed at some point between (400,2700]. The nominal reliability calculations in Ref. 6 are based on correcting all BDD modes at 400 hours.

Table 1. Failure Times in First 400 Hours of Test, Failure Mode Status, and Failure Mode Name

Time to Failure	Failure Mode Status	Failure Mode Name	Time to Failure	Failure Mode Status	Failure Mode Name
0.7	BDC	1	192.7	BDD	12
15	BDD	2	213	A	35
17.6	BC	23	244.8	A	36
25.3	BDD	3	249	BDD	13
47.5	BDD	4	250.8	A	35
54	BDD	5	260.1	BDD	2
54.5	BC	24	273.1	A	35
56.4	BDD	6	274.7	BDD	6
63.6	A	34	282.8	BC	32
72.2	BDD	5	285	BDD	14
99.2	BC	25	315.4	BDD	4
99.6	BDD	7	317.1	A	34
100.3	BDD	8	320.6	A	36
102.5	A	34	324.5	BDD	12
112	BDC	9	324.9	BDC	10
112.2	BC	26	342	BDD	5
120.9	BDD	2	350.2	BDD	3
121.9	BC	27	355.2	BC	33
125.5	BDC	10	364.6	BDC	10
133.4	BDD	11	364.9	A	35
151	BC	28	366.3	BDD	2
163	BC	29	379.4	BDC	15
174.5	BC	30	389	BDC	16
177.4	BDC	10	394.9	A	36
191.6	BC	31	395.2	BDD	17

The Extended Continuous Evaluation model in Ref. 6 estimates: Demonstrated MTBF = 6.54, Growth Potential MTBF = 12.8, Phase 2 Projected MTBF = 10.4, lambda discovery = 0.4517, and beta discovery = 0.6055. From Ref. 6 the average nominal FEF = 0.699. Although the Phase 2 projected MTBF is above the target of 9.7 the estimated growth potential of 12.8 is less than the assumed 14.37 used in the planning model. Can Contractor 1 still expect to attain the requirement of 11.5 at 2700 hours? Applying the multi-phase projections in eq. 17 we estimate from data: Phase 2 MTBF = 10.4, Phase 3 MTBF = 11.2, and at 2700, the end of test, MTBF =11.5. The multi-phase projections suggest that there should be no changes in the reliability growth program and indicates that the system is capable of attaining the goal at 2700 hours if the present failure mode discovery and corrective action effort is maintained.

6.2 Example 2

Suppose Contractor XYZ used the PM 2 model and only considered 400 hours of testing for reliability growth. Information based on a Management Strategy of 0.95, FEF = 0.70, Growth Potential = 14.37 and a requirement of 11.5, were entered into the PM2 model according to Ref. 8. The model will show a curve to the goal of 11.5 for any test time T, in this case T = 400 hours, as shown in Figure 2.

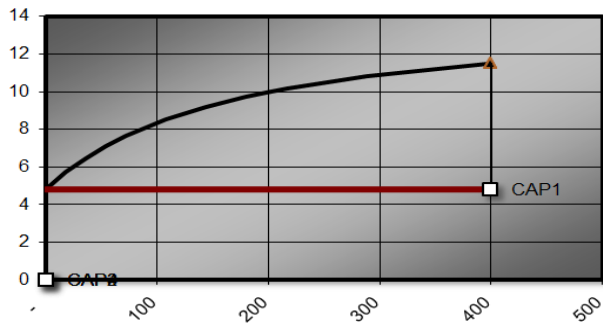


Figure 2. PM 2 Planning Model Growth Curve

Contractor XYZ then runs the 400 reliability growth test and analyzes the data, given in Table 1. Based on the projection of 10.4 Contractor XYZ concludes that additional test time is needed. Contractor XYZ then uses eq. 18 to construct a new Extended Model Plan Growth Curve based on data to $T = 400$, Growth Potential = 12.8, FEF = 0.699, $\lambda_D = 0.4517$, $\beta_D = 0.6055$. See Fig. 3. The calculations show that the 11.5 goal could be reached as early as $T = 2400$ hours. The first 400 hours have been completed and the 10.4 MTBF on this idealized growth curve has already been achieved. Contractor XYZ then adds an additional 2000 hours of testing for reliability growth utilizing existing testing scheduled for functionality and performance. The new idealized planned growth curve is reflected in Fig. 3.

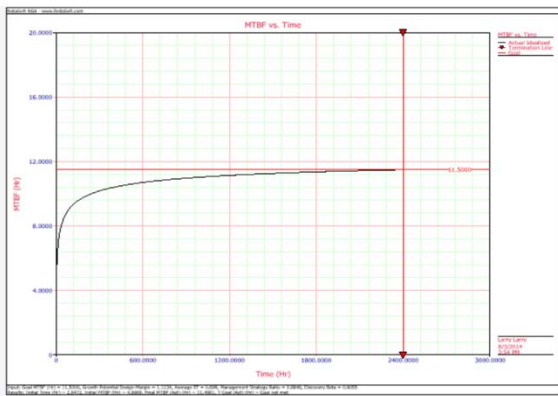


Figure 3. New Extended Model Planned Growth Curve Based on Data to $T = 400$ Hours.

REFERENCES

1. J. D. Selby and S.G. Miller, Reliability Planning and Management (RPM), Paper Presented at ASQC/SRE Seminar, September 1970.

2. J. T. Duane, Learning Curve Approach to Reliability Monitoring, IEEE Transactions on Aerospace. Vol. 2, 1964, PP.563-566.
3. Reliability Growth Management, Department of Defense Military Handbook 189; Naval Publications and Form Center, Philadelphia, 1981.
4. L. H. Crow, Planning a Reliability Growth Program Utilizing Historical Data, Proceedings of the 2011 Annual RAMS Symposium, January 2011, Orlando, FL
5. L. H. Crow, Reliability Growth Projection from Delayed Fixes, Proceedings 1983 Annual Reliability and Maintainability Symposium, pp. 84-89, Orlando, FL.
6. L. H. Crow, The Extended Continuous Evaluation Reliability Growth Model, Proceedings of the 2010 Annual RAMS Symposium, January 2010, San Jose, CA.
7. M. Krasich, Reliability Growth Testing, What is the Real Final Result?, Proceedings of the 2014 Annual RAMS Symposium, January 2011, Colorado Springs, CO.
8. Reliability Growth Management, Department of Defense Military Handbook 189A; 10 Sept 2009.

BIOGRAPHY

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Dr. Larry H. Crow is president of CRR. Previously Dr. Crow was VP, Reliability & Sustainment Programs, at ALION Science and Technology, Huntsville, AL. From 1985 to 2000, Dr. Crow was Director, Reliability, at General Dynamics ATS -- formally Bell Labs ATS. From 1971-1985, Dr. Crow was chief of the Reliability Methodology Office at the US Army Materiel Systems Analysis Activity (AMSAA). He developed the Crow (AMSAA) reliability growth model, which has been incorporated into US DoD handbooks, and national & international standards. He chaired the committee to develop Mil-Hdbk-189, 1981, *Reliability Growth Management*, and is the principal author of that document. Dr. Crow is a Fellow of the American Statistical Association, and the Institute of Environmental Sciences and Technology. He is a Florida State University Alumni Association Distinguished Alumnus and the recipient of the FSU "Grad Made Good" Award for the Year 2000, the highest honor given to a graduate by Florida State University