On Planning Accelerated Life Tests for Comparing Two Product Designs

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SUMMARY & CONCLUSIONS

Accelerated life test (ALT) planning is one of the most important and challenging tasks for reliability engineers. Since the late 1970s, methods for efficient ALT planning have been studied extensively and over 150 research papers have been published [1]. Most of the existing methods focus on designing tests to minimize the estimation precision of model parameters or their functions. Popularly used test designs such as the 2-level statistically optimum plan, 3-level best compromise plan and 3-level best standard plan are all based on this theory. However, although these designs are very useful for estimating distribution parameters or given reliability metrics, they are not efficient for planning tests that compare different products.

In this paper, we will present two methods for designing ALT to compare two different designs in terms of their B10 life. The probability of detecting a given amount of difference of the B10 lives is the focus of the proposed methods. This probability usually is called detection power. Comparing the estimated lives of two designs is the same as comparing two random variables since each life estimated through the ALT data is a random variable. According to the required detection probability, the sample size of a comparison test can be determined by either the analytical or the simulation method given in this paper. An example is used in the paper to illustrate the theory and the applications of the proposed methods. The presented methods are general methods and can be extended to other situations and applied beyond the example used in this paper.

1 PROBLEM STATEMENT

Assume the life of a product is affected by voltage. High voltage will shorten the life and the usage level voltage is 70 volts. Accelerated life tests are conducted for one design option of the product and the test results are given in Table 1. For convenience, we'll refer to this design as design A.

F/S	Time	Voltage	F/S	Time	Voltage
F	550	120	F	871	95
F	700	120	F	889	95
F	856	120	F	894	95
F	968	120	F	954	95
F	995	120	F	1022	95
F	1051	120	F	1030	95
F	1072	120	F	1220	95
F	1168	120	F	1228	95
F	1280	120	F	1246	95
F	1321	120	F	1363	95
F	1436	120	F	1607	95
F	1445	120	F	1680	95
F	1588	120	F	1701	95
F	1654	120	F	1830	95
F	1786	120	F	1895	95
F	1802	120	F	2180	95
F	1863	120	F	2326	95
F	1937	120	F	2384	95
F	2168	120	F	2446	95
F	2334	120	F	2531	95
F	546	95	F	2562	95
F	623	95	F	2612	95
F	656	95	F	2678	95
F	664	95	F	2891	95
F	771	95	S	3000	95
F	775	95	S	3000	95
F	779	95	S	3000	95

Table 1- Test Data for Design A at 120v and 95v

From Table 1, we can see that 20 samples were tested at 120v and 34 were tested at 95v. Using these data, we fit a life-stress model and predict the B10 life at the usage level. The model and the prediction are given in the following section.

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1.1 Accelerated Life Test Data Analysis

The inverse power law-Weibull (IPL-Weibull) model was used here. The cumulative distribution function (cdf) of it is:

$$F(t,V) = 1 - e^{-\left(KV^{n}t\right)^{\beta}} \quad (1)$$

where t is life time; V is voltage; K and n are model parameters in the life-stress relation function; and β is the shape parameter of the Weibull distribution. For data in Table 1, the model parameters calculated using maximum likelihood estimation (MLE) are: K=2.6282×10^{-6}, n=1.1562 and β=2.3240. Applying this model, the predicted B10 life at the usage level of 70v is 1,063 hours. Its one-sided upper bound at a confidence level of 90% is 1,478. (For details on accelerated life data modeling and parameter estimation, please refer to [2, 3].) However, the required B10 life at the use stress level should be at least 1,500 hours. Clearly, the current design couldn't meet this requirement. Therefore, this design is modified. The modified design is called design B. Due to cost and time constraints, only 20 samples of design B were manufactured and tested for 3,000 hours at 120v in order to quickly show the improvement. The test data at 120v is given in Table 2.

F/S	Time	Voltage	F/S	Time	Voltage
F	876	120	F	2159	120
F	1023	120	F	2207	120
F	1345	120	F	2412	120
F	1473	120	F	2693	120
F	1527	120	F	2721	120
F	1580	120	F	2725	120
F	1638	120	F	2826	120
F	1796	120	F	2926	120
F	1924	120	S	3000	120
F	1934	120	S	3000	120

Table 2- Test Data for Design B at 120v

After applying the Weibull distribution to data in Table 2, the calculated B10 life for design B at 120v is 1,194. If we also fit a Weibull distribution to the failures for design A at 120v in Table 1, the predicted B10 life is 782. Therefore, the new design shows significant improvement at a voltage level of 120v. The ratio of these two B10 lives is about 1.525. Will this ratio still hold for the B10 life at the usage voltage level? If so, the new design will meet the reliability requirement of a B10 life of 1,500 hours. In order to answer this question, test data at another stress level of design B are needed. Test engineers decided to test more samples at a voltage level of 95v. The question is how many samples are needed? The engineers only have 5,000 hours for testing, and manufacturing test samples is costly. Due to these constraints, the engineers want to build and test as few samples as possible.

The engineers decided to determine the sample size based on the requirement of the detection probability. For design A, we know that the predicted B10 life at the usage stress level is 1,063 with a one-sided 90% upper bound of 1,478. For design B, the B10 life will be predicted using data at a voltage level of 120v (Table 2) and data at a voltage level of 95v (to be tested). The requirement for the detection probability is to have at least an 80% chance to detect this difference, if the ratio of the two B10 lives is at least 1.5. In other words, hopefully through choosing the right sample size to be tested at 95v for design B, we will have an 80% chance of detecting the difference of the two B10 lives. Statistically speaking, the requirement of planning the test is that the detection probability must be at least 80% at a confidence level of 90% when the ratio of the B10 lives of the two designs is 1.5.

In section 2, we will illustrate the theory of determining the minimal sample size based on the required detection probability.

2 AN ANALYTICAL SOLUTION

2.1 Calculate the Variance of the B10 Life

The B10 life is the 90% percentile of the failure time distribution. If we define x as the log transform of B10 life, it can be calculated by:

$$\begin{aligned} x &= \frac{\ln\left(-\ln(0.9)\right)}{\beta} - \ln(K) - n\ln(V) \\ &= \frac{-2.2504}{\beta} - \ln(K) - n\ln(V) \end{aligned} \quad (2)$$

Using the delta method [4, 6], the variance of x is calculated by:

$$Var(x) = \sum_{i=1}^{3}\left(\frac{\partial x}{\partial \theta_i}\right)^2 Var(\theta_i) + \sum_{i=1}^{3}\sum_{j=1, j\neq i}^{3} \frac{\partial x}{\partial \theta_i}\frac{\partial x}{\partial \theta_j} Cov(\theta_i, \theta_j) \quad (3)$$

where θ_i is model parameter β, K and n. $Var(\theta_i)$ and $Cov(\theta_i, \theta_j)$ are obtained from the Fisher information matrix of the failure data [4, 6]. Once the predicted value of x and its variance are obtained using Eq. (2) and (3), the confidence bounds of x can be calculated by assuming that x is normally distributed with the predicted mean and variance.

2.2 Calculate the Detection Probability

For the design A data in Table 1, the predicted B10 life at 70v is 1,063. Therefore its log transform is x^A=ln(B10)=ln(1,063)=6.9689. The calculated variance of x^A is 0.066, so the standard deviation $std(x^A)$ is 0.2571 by taking the square root of the variance. x^A usually is assumed to be normally distributed. This assumption has been proven by simulation studies and is used by [4, 5]. The one-sided upper bound of x^A is:

$$x_U^A = x^A + Z_{CL=0.9}\,std(x^A) = 7.2984;$$

where $Z_{Cl=0.9}$ is the 90% percentile of the standard normal distribution.

If the B10 life of design B is 1.5 times the B10 life of design A, the log of the B10 life will be x^B= x^A+ln(1.5)=6.9689+0.4055=7.3744. In other words, the predicted x^B at 70v of design B will be a random number following a normal distribution with mean of 7.3744. Its standard deviation is determined by the test data at 120v (Table 2) and 95v (to be tested). If the calculated x^B is larger than x_U^A the upper bound of x^A at a confidence level of *CL*, we conclude that the B10 life of design B is larger than design A at a confidence level of *CL*. Therefore, we have detected the difference of the B10 lives. The probability that the calculated x^B is larger than x_U^A is:

$$\begin{aligned} &\text{Detection Probability} = \Pr(x_U^A < x^B) = \Pr(7.2984 < x^B) \\ &= \Pr\left(\frac{7.2984 - 7.3744}{std(x^B)} < z\right) = 1 - \Phi_{norm}\left(\frac{-0.076}{std(x^B)}\right) \geq 0.8 \end{aligned} \tag{4}$$

where z is the standard normal random variable. Φ_{norm} is the cdf of the standard normal distribution. The calculated detection probability should be larger than the required value of 0.8. In order to meet this requirement, from Eq. (4) we calculate that $std(x^B)$ must be less than 0.0903. This standard deviation is used to determine the sample size at 95v for design B.

2.3 Determine the Sample Size

2.3.1 Theory of Fisher Information Matrix

From the theory of Fisher information matrix, we know the standard deviation of the estimated distribution parameters and the function of them are affected by the test sample size. Therefore, we can determine the sample size of the test at 90v for design B based on the solved standard deviation in section 2.2. First, let's discuss how the Fisher information matrix is developed when maximum likelihood estimation is used.

It is known that if failure time *t* has a Weibull distribution with parameters of β and η, the natural logarithm *y*=ln(*t*) of it has a smallest extreme value (SEV) distribution. The cdf for SEV is

$$F(y) = 1 - e^{-e^{(y-\mu)/\sigma}} \tag{5}$$

where $\mu = \ln(\eta)$; $\sigma = 1/\beta$.

For convenience, the IPL life-stress function used in Eq. (1) is re-parameterized as:

$$\begin{aligned} \eta_V &= \frac{1}{KV^n} \Rightarrow \ln(\eta_V) = -\ln(K) - n\ln(V) \\ &\Rightarrow \mu_V = \alpha_0 + \alpha_1 S \end{aligned} \tag{6}$$

where $\mu_V = \ln(\eta_V)$, $-\ln(K) = \alpha_0$, $\alpha_1 = -n$, $S = \ln(V)$.

Let $z = (y - \mu_V)/\sigma$ for failure times, and for the suspension time, we define

$$\Phi_{sev}(w) = \Phi_{sev}\left(\frac{\ln(T) - \alpha_0 - \alpha_1 S}{\sigma}\right)$$

where *T* is the suspension time. Φ_{sev} is the cdf of the standard SEV distribution. The log likelihood function is:

$$\Lambda = \sum_{i=1}^{l}\sum_{j=1}^{N_i^f}\left(-\ln(\sigma) - e^{z_{i,j}} + z_{i,j}\right) + \sum_{i=1}^{l}\sum_{j=1}^{N_i^s}\ln\left(1 - \Phi_{sev}(w_{i,j})\right) \tag{7}$$

where $z_{i,j}$ is from the *j*th failure time at the *i*th stress; *l* is the total number of stresses; N_i^f is the number of failures at the *i*th stress; N_i^s is the number of suspensions at the *i*th stress; $w_{i,j}$ is for the *j*th suspension time at the *i*th stress.

From Eq. (7), the Fisher information matrix for the test data in terms of σ, α_0 and α_1 is:

$$F = -\begin{bmatrix} E\left(\frac{\partial^2\Lambda}{\partial\sigma^2}\right) & E\left(\frac{\partial^2\Lambda}{\partial\sigma\partial\alpha_0}\right) & E\left(\frac{\partial^2\Lambda}{\partial\sigma\partial\alpha_1}\right) \\ E\left(\frac{\partial^2\Lambda}{\partial\sigma\partial\alpha_0}\right) & E\left(\frac{\partial^2\Lambda}{\partial\alpha_0^2}\right) & E\left(\frac{\partial^2\Lambda}{\partial\alpha_0\partial\alpha_1}\right) \\ E\left(\frac{\partial^2\Lambda}{\partial\sigma\partial\alpha_1}\right) & E\left(\frac{\partial^2\Lambda}{\partial\alpha_0\partial\alpha_1}\right) & E\left(\frac{\partial^2\Lambda}{\partial\alpha_1^2}\right) \end{bmatrix} \tag{8}$$

For the calculation of each element in Eq. (7), please refer to Appendix and [6]. The asymptotic covariance matrix of the estimated distribution parameters is the inverse of the Fisher information matrix in Eq. (8). That is,

$$\Sigma = \begin{bmatrix} Var(\sigma) & Cov(\sigma,\alpha_0) & Cov(\sigma,\alpha_1) \\ Cov(\sigma,\alpha_0) & Var(\alpha_0) & Cov(\alpha_0,\alpha_1) \\ Cov(\sigma,\alpha_1) & Cov(\alpha_0,\alpha_1) & Var(\alpha_1) \end{bmatrix} = F^{-1} \tag{9}$$

2.3.2 Calculate Fisher Information Matrix for Design B

For design B, since there are no test data at 95v yet, we cannot estimate the model parameters and calculate the local Fisher information based on failure data and the estimated parameters. However, we can use the information from design A to calculate the planning value of the model parameters for design B. From the current test results, it is known the failure mechanism under study is the same for both designs. Therefore, we assume the IPL-Weibull model is also applicable to design B, and β and n in the model are the same for both designs. The difference of the life is only reflected by parameter *K*. Since we are interested in the situation where the B10 life of design B is 1.5 times the B10 life of design A, this leads us to set *K* for design B to be 1.5 times smaller than the *K* for design A. Therefore, using the estimated parameters of design A in section 1.1, the planning values for design B are:

$$\beta = 2.324;\ K_B = K_A/1.5 = 1.75E-6;\ n = 1.1562,$$

or in terms of σ, α_0 and α_1:

$$\sigma = 0.4303;\ \alpha_0 = 13.2547;\ \alpha_1 = -1.1562$$

Using the above planning values in Eq. (7) we can calculate the Fisher information matrix for design B for the data in Table 2. It is,

$$F_{120} = \begin{bmatrix} 134.46 & 18.94 & 90.66 \\ 18.94 & 104.47 & 500.14 \\ 90.66 & 500.14 & 2394.40 \end{bmatrix} \tag{10}$$

F_{120} will be combined with F_{95}, the one we will discuss shortly, to get the total Fisher information matrix. Since no data were available yet for the test at 95v, we have to use the *expected* Fisher information. For any test sample at 95v, it would either fail or survive by the end of a test time of T=5,000. Let

$$I = \begin{cases} 1 & \text{if a test unit fails} \\ 0 & \text{if a test unit survives} \end{cases}$$

Then the log likelihood Λ_j for the jth test unit at stress S is,

$$\Lambda_j = I\left(-\ln(\sigma) - e^{z_j} + z_j\right) + (1-I)\ln\left(1-\Phi_{sev}(w)\right) \quad (11)$$

For all the m test units, the log likelihood function is $\Lambda = \sum_{j=1}^{m} \Lambda_j$. Since we don't yet have observations at 95v, F_{95} is calculated using the planning model parameters and the expected value of the derivatives in Eq. (8). For details of the calculation, please see Eq. (A.2) in Appendix and [6]. The expected Fisher information matrix for the m test units at 95v is:

$$F_{95} = m\begin{bmatrix} 8.7578 & 1.8436 & 8.3957 \\ 1.8436 & 5.2221 & 23.7808 \\ 8.3957 & 23.7808 & 108.2950 \end{bmatrix} \quad (12)$$

The total Fisher information matrix for design B is

$$F = F_{120} + F_{95} \quad (13)$$

m is the only unknown in Eq. (13) and it can be solved by setting the calculated standard deviation $std(x^B)$ to be equal to 0.0903, the standard deviation value obtained based on the required detection probability in section 2.2. The calculated value of m is 114. When m=114, the total F is:

$$F = \begin{bmatrix} 1132.8444 & 229.1131 & 1047.7772 \\ 229.1131 & 699.7880 & 3211.1535 \\ 1047.7772 & 3211.1535 & 14740.0373 \end{bmatrix}$$

The variance and covariance matrix is

$$\Sigma = F^{-1} = \begin{bmatrix} 0.0009 & -0.0035 & 0.0007 \\ -0.0035 & 4.3557 & -0.9487 \\ 0.0007 & -0.9487 & 0.2067 \end{bmatrix}$$

From Eq. (2), we know x^B is calculated by

$$x^B = \frac{-2.2504}{\beta} - \ln(K) - n\ln(95)$$
$$= -2.2504\sigma + \alpha_0 + \alpha_1 4.5539$$

Define L = (-2.2504, 1, 4.5539), the variance of x^B and the standard deviation $std(x^B)$ are calculated by

$$Var(x^B) = L \times \Sigma \times L' = 0.0081\text{; } std(x^B) = \sqrt{Var(x^B)} = 0.0900$$

It confirms that the calculated $std(x^B)$ is less than the required value of 0.0903 when 114 samples are tested. If we use 113 samples, the predicted standard deviation $std(x^B)$ is 0.9037 which is slightly larger than the required value. Therefore at least 114 units should be tested at 95v for design B in order to have an 80% chance of detecting the difference of the B10 lives.

3 A SIMULATION APPROACH

In section 2, an analytical solution was found for determining the test sample size in order to meet the detection probability requirement. In this section we will discuss a novel simulation solution by generating the expected failure times at 95v. Similar to the assumptions used in section 2, we assume the failure times at 95v follow a Weibull distribution with the following planning values as given in section 2:

$$\beta = 2.324\text{; } K_B = K_A / 1.5 = 1.75E-6\text{; } n = 1.1562$$

From the above planning values, we can solve the scale parameter $\eta = 1/(KV^n) = 2950.1819$. Let's assume m samples are tested. When the ith failure occurs, the estimated probability of failure using the approximated median rank method is [6]:

$$F_i = (i-0.3)/(m+0.4) \quad (14)$$

From Eq. (14) the expected failure time t_i for the ith failure is solved from:

$$F_i = 1 - e^{(t_i/\eta)^\beta} \quad (15)$$

For example, when 10 samples are tested, the estimated median rank for the 1st failure is:

$$F_1 = (1-0.3)/(10+0.4) = 0.0673$$

Setting Eq. (15) equal to 0.0673, the expected failure time for the 1st failure of the 10 samples is calculated to be 937.6604 using the planning values for β and η. With the calculated failure time for each failure from Eq. (15) and the planning values for model parameters, we can get the Fisher information matrix, variance/covariance matrix and the expected standard deviation for the B10 life. If the calculated standard deviation is bigger than the required value, we need to repeat the above procedure by increasing the sample size until the one that meets the requirement is found.

From Section 2, we know the analytical solution for the sample size is 114. Using the above simulation procedure with a sample size of 114, we can find the expected failure time for each sample using Eq.(15). Since the available test time is 5,000 hours, any failures occur beyond 5,000 are treated as suspension. The expected failure times of the 114 samples are given in Table 3.

Sample	Median Rank	Expected Failure Time	Failure /Suspension
1	0.0061	329.6411	**F**
2	0.0149	483.8150	**F**
3	0.0236	591.5095	**F**
4	0.0323	678.7011	**F**
5	0.0411	753.7494	**F**
6	0.0498	820.5871	**F**
7	0.0586	881.4279	**F**
8	0.0673	937.6604	**F**
…	…	…	…
113	0.9851	5,000	**S**
114	0.9939	5,000	**S**

Table 3- Expected Results of 114 Test Samples for Design B at 95v

Using the data in Table 3 and the planning parameters given in section 2.3.2 in terms of σ, α_0 and α_1 we got the expected Fisher information matrix

$$F_{95} = \begin{bmatrix} 993.3886 & 206.4769 & 940.2706 \\ 206.4769 & 593.7927 & 2704.0588 \\ 940.2706 & 2704.0588 & 1231.9508 \end{bmatrix} \quad (16)$$

Comparing Eq. (16) with the analytical solution of Eq. (12) by setting m=114, the relative differences for all the elements in these two matrixes can be calculated as:

$$D_{Fisher\_95} = \begin{bmatrix} 0.38\% & 0.89\% & 0.89\% \\ 0.89\% & 0.63\% & 0.63\% \\ 0.89\% & 0.63\% & 0.63\% \end{bmatrix} \quad (17)$$

From Eq. (17) it can be seen that the simulation approach for obtaining expected failure times provides a result that is very close to the analytical solution given in section 2. Similar to the analytical solution, we can get the expected value of the total Fisher information matrix by combining Eq. (10) and Eq. (16). From here the expected variance/covariance matrix can be calculated and the expected value of $std(x^B)$ is calculated to be 0.08998. This result is almost identical to the analytical solution of 0.0900.

Therefore, both the analytical and the simulation methods suggest that at least 114 samples should be tested at 95v for design B in order to have an 80% chance of detecting the difference of the B10 lives of design A and design B.

4 CONCLUSIONS

In this paper, we presented two methods for determining sample size according to the required detection probability of an accelerated life test for comparing two designs. An example is used to illustrate the theory of these two methods. Although part of the test data (test at 120v) of design B and all the test data of design A have been given in this example, the proposed method can be applied to cases where no data are available. As long as the planning values and the detection probability are provided and the improved design is expected to have the same failure mechanisms as the old design, the methods presented in this paper can be applied. Any unknown variable in the test plan such as the stress level, sample size at each stress level, and the test time can be solved given that other variables are provided. The analytical solution provided in this paper requires numerically solving the expected Fisher information matrix. For engineers with a strong math background, this method may be faster than the simulation method since no trial and error is required to find the suitable sample size. With the simulation method, once the failure times are simulated, the Fisher information matrix can be easily calculated using equations given in the Appendix. Because this is a very straightforward method, it will often be easier than the analytical method for engineers to understand and use.

5 APPENDIX

From Eq. (11), the six second partial derivatives in the Fisher information matrix are:

$$\frac{\partial^2 \Lambda}{\partial \alpha_j \partial \alpha_k} = \left(-\frac{g_j g_k}{\sigma^2}\right)\left[Ie^z + (1-I)e^w\right]; \quad j,k = 0,1$$

$$\frac{\partial^2 \Lambda}{\partial \alpha_j \partial \sigma} = \left(-\frac{g_j}{\sigma^2}\right)\left[Ize^z + \left(1-I\right)we^w\right]; \quad j = 0,1 \qquad \text{(A.1)}$$

$$\frac{\partial^2 \Lambda}{\partial \sigma^2} = \left(-\frac{1}{\sigma^2}\right)\left[I\left(1+z^2e^z\right) + \left(1-I\right)w^2e^w\right]$$

where $g_0 = 1$, $g_1 = S = \ln(V)$, and

$$w = \frac{\ln(T) - \alpha_0 - \alpha_1 S}{\sigma}$$

For an observation, either it is a failure or a suspension, Eq. (A.1) can be applied directly by setting I=1 or 0. It is used to get the Fisher information matrix for data at 120v and the simulated failure data at 95v in the simulation method for design B.

If there are no observations yet, the expected Fisher information matrix can be calculated by taking the expectation of the terms in Eq. (A.1). They are:

$$E\left[\frac{\partial^2 \Lambda}{\partial \alpha_j \partial \alpha_k}\right] = \left(-\frac{g_j g_k}{\sigma^2}\right)\left[\Phi_{sev}(w)\right]; \quad j,k = 0,1$$

$$E\left[\frac{\partial^2 \Lambda}{\partial \alpha_j \partial \sigma}\right] = \left(-\frac{g_j}{\sigma^2}\right)\left[\int_0^{e^w} \ln(x)xe^{-x}dx + \left(1-\Phi_{sev}(w)\right)we^w\right]; \quad j = 0,1$$

$$E\left[\frac{\partial^2 \Lambda}{\partial \sigma^2}\right] = \left(-\frac{1}{\sigma^2}\right)\left[\Phi_{sev}(w) + \int_0^{e^w} \ln^2(x)xe^{-x}dx + \left(1-\Phi_{sev}(w)\right)w^2e^w\right] \qquad \text{(A.2)}$$

Eq. (A.2) is used to calculate the expected Fisher information matrix for the planned samples at 95v for design B in the analytical method.

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Adamantios Mettas is the VP of Product Development at ReliaSoft Corporation, spearheading ReliaSoft's innovative product development group. In his previous position as ReliaSoft's Senior Scientist, he played a key role in the development of ReliaSoft's software including Weibull++, ALTA, RGA and BlockSim by deriving statistical formulations and models and developing numerical methods for solving them. Mr. Mettas has trained more than 2,000 engineers throughout the world on the subjects of Life Data Analysis, FMEA, Warranty Analysis, System Reliability, RCM, DOE and Design for Reliability, as well as advanced topics such as Accelerated Testing, Repairable Systems and Reliability Growth. He has published numerous papers on these topics. In addition to training, Mr. Mettas is part of ReliaSoft's consulting arm and has been involved in various projects across different industries, including Oil & Gas, Power Generation, Automotive, Semiconductor, Defense and Aerospace. Mr. Mettas holds an M.S. in Reliability Engineering from the University of Arizona and he is a Certified Reliability Professional (CRP).

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