# Reliability Estimation for One-Shot Systems with Zero Component Test Failures

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*SUMMARY & CONCLUSIONS*

Tests for one-shot systems such as missiles and rockets are very expensive. In order to design an efficient test plan to demonstrate the required reliability in the final system test, system reliability should be studied in advance. Before the final system tests, many subsystem level tests usually have already been conducted by customers and manufacturers. Therefore, the system reliability can be estimated using the information obtained from these tests before the final system test. Due to the highly reliable nature of one-shot systems, it is very unlikely to observe many failures, even at the subsystem level tests. To accurately estimate the system reliability with few failures or even without failures is very challenging. A lot of research has been done on how to estimate the system reliability and its confidence intervals from its subsystem test data. However, most of them require failures at the sub-level tests. When there are no failures, these methods do not work.

In this paper, a flexible and practical method is designed to estimate the system reliability and its confidence bounds when there are few or no failures during the subsystem tests. This method can be applied to series, parallel and complex systems. The estimated system reliability information is then used to design an efficient test plan for the final system reliability demonstration test. A case study shows that the proposed method is very efficient and accurate when compared with existing methods and simulation results.

## *1 INTRODUCTION*

The test for a one-shot system is a Bernoulli trial and the number of successes follows a Binomial distribution. Based on the Binomial formula, a test can be designed to demonstrate the reliability of a one-shot system at a given confidence level [1]. This is called design of reliability tests, or DRT in short. However this traditional method used in planning the test ignores the information from the subsystem tests. Therefore, an overly conservative result usually is obtained and a large sample size is needed in order to demonstrate a required reliability. In the following sections of this paper, the terms "subsystem" and "component" are used interchangeably.

For a system the reliability information of its subsystems is usually available before the final system demonstration tests. If this information can be considered in the test plan, a more efficient test can be designed. Much research on how to estimate system reliability from component reliabilities has been done [2-9]. These methods can be classified into three categories:

- Approximated analytical method [2-4, 6]
- Simulation method [4-6, 9]
- Bayesian method [7-9]

The variance decomposition method is one of the most popular analytical methods. Coit [2] and Jin and Coit [3] used the estimated means and the variances of component reliabilities to get the mean and variance of the system reliability first, and then computed the confidence interval for the system reliability by assuming the system reliability is lognormally distributed. In their method, the mean and variance for each component are calculated by:

$$\hat{r}_i = \frac{x_i}{n_i}; \qquad (1)$$

$$Var(\hat{r}_i) = \frac{\hat{r}_i(1-\hat{r}_i)}{n_i} \qquad (2)$$

where $n_i$ is the number of tests for the $i$th component; $x_i$ is the number of successes and $\hat{r}_i$ is the estimated reliability. Equation (2) fails when there are no failures at the component tests. Other analytical methods for estimating system reliability from component data can be found in [4]. Most of them cannot be used for zero failure tests or can only be used for series systems.

Bayesian theory was also applied in the system reliability estimation in the last two decades [7-9]. Although the Bayesian method can handle the non-failure situation, it is cumbersome for practical use and the results are affected by the assumed prior distribution. Martz, Waller and Fickas used the beta distribution as the prior distribution for subsystems [7]. The prior distributions are modified by the subsystem test data to get the posterior distributions for subsystem reliability. These posterior distributions are then used as the "induced" prior to get the posterior distribution for the system reliability. Once the system reliability distribution is obtained, its confidence interval and percentiles can be calculated.

However, the calculation of this method is tedious and approximation is required when applied for complex systems.

In additional to the above analytical solutions, bootstrap estimation is also studied in recent years [5, 6, 9]. In the use of simulation, for a zero failure subsystem, its reliability is either simulated from an assumed beta prior distribution [5], or using $r_i = 1-\varepsilon$ for some small value of $\varepsilon$ [9]. The quality of the results depends on the assumptions used in the simulation.

In this paper, a new and practical method to obtain the system reliability from zero failure component tests is proposed. The proposed method integrates the advantages of the existing methods and is easy to use. The remainder of this paper is organized as follows. In section 2, we review several commonly used methods for calculating the confidence intervals for component/system reliability. The Clopper-Pearson method will be discussed in detail [10]. In section 3, we prove that the Clopper-Pearson method in fact is a special case of the Bayesian method when a special non informative prior distribution is used. In section 4 a procedure of getting the system reliability and its bounds from the component test data is proposed. Section 5 gives equations on how to design a demonstration test using the estimated system reliability. In section 6, an example is used to illustrate the proposed method in this paper. Finally, conclusions are given in section 7.

## *2 CONFIDENCE INTERVAL FOR THE PROBABILITY OF SUCCESS*

Assuming the probability of success is $p$, the confidence interval (CI) for $p$ has been studied extensively. The simplest and most commonly used formula is the normal distribution approximation. The formula for this method is [11]:

$$\hat{p} \pm z_{1-\alpha/2}\sqrt{\left[\hat{p}(1-\hat{p})\right]/n} \tag{3}$$

where, $z_{1-\alpha/2}$ is the $1-\alpha/2$ percentile of the standard normal distribution, and $n$ is the sample size. $\hat{p}$, the ML (maximum likelihood) estimate of $p$. $\hat{p}$ is calculated by dividing the number of success by the sample size. However, equation (3) does not work when $\hat{p}$ is 0 or 1. To overcome the above drawback, some other methods have been proposed [10, 12-14]. Among these methods, the Wilson score interval [12] and the Clopper-Pearson interval [10] are the most popular two. The Wilson score interval is:

$$\frac{\hat{p}+\frac{1}{2n}z_{1-\alpha/2}^2 \pm z_{1-\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}+\frac{z_{1-\alpha/2}^2}{4n^2}}}{1+\frac{1}{n}z_{1-\alpha/2}^2} \tag{4}$$

Equation (4) works for a small number of trials and extreme values of $\hat{p}$.

Clopper-Pearson [10] developed an exact method by using the binomial distribution directly. The Clopper-Pearson interval can be written as:

$$\{p \mid P[Bin(n;p) \le x] \ge \alpha/2\} \cap \{p \mid P[Bin(n;p) \ge x] \ge \alpha/2\} \tag{5}$$

where $x$ is the number of successes and $Bin(n;p)$ is a binomial random variable with $n$ trials and probability of success of $p$. Equation (5) usually is re-written as [5]:

$$\sum_{k=x}^{n}\binom{n}{k}p_L^k(1-p_L)^{n-k} = \alpha/2 \tag{6.1}$$

$$\sum_{k=0}^{x}\binom{n}{k}p_U^k(1-p_U)^{n-k} = \alpha/2 \tag{6.2}$$

where $p_L$ and $p_U$ are the lower and upper 2-sided confidence bounds at a confidence level of 1-*α* for *p*. For one-sided confidence bound, we only need to change *α*/2 to *α* in equations (6.1) and (6.2).

Because of the relationship between the cumulative binomial probability function, the beta distribution and the *F*-distribution, equations (6.1) and (6.2) can be calculated using [5, 15]:

$$\frac{1}{1+\frac{n-x+1}{xF_{2x,2(n-x+1),1-\alpha/2}}} \le p \le \frac{1}{1+\frac{n-x}{(x+1)F_{2(x+1),2(n-x),\alpha/2}}} \tag{7}$$

The Clopper-Pearson bound is also known as the beta-binomial bound or the "exact" bound because there are no assumptions on $p$ [1, 15].

Equation (3) assumes $p$ is normally distributed with a mean of $\hat{p}$ and a variance of $\left[\hat{p}(1-\hat{p})\right]/n$, while a similar assumption is also used in equation (4). In fact, with the test data available, the exact distribution of $p$ can be obtained. In section 3, we will provide the exact distribution when there are no failures in the test.

## *3 THE EXACT DISTRIBUTION OF THE RELIABILITY*

The well-known beta-binomial one-sided lower bound for the reliability (probability of success) of a system/component can be calculated by [1]:

$$\sum_{k=0}^{y}\binom{n}{k}(1-r_L)^k r_L^{\,n-k} = 1-CL = \alpha \tag{8}$$

where *CL* is the confidence level, $r_L$ is the lower bound for the reliability, $n$ is the sample size and $y$ is the number of failures. In fact, it can be seen that equation (8) is the same as equation (6.1). When there are no failures, both equations (8) and (6.1) become:

$$1-CL = r_L^n \tag{9}$$

Based on the definition of the confidence level, equation (9) can be written as:

$$1-CL = \Pr(r \le r_L) = F(r_L) = r_L^n \tag{10}$$

where $r$ is the random variable for reliability. $F(\cdot)$ is the *cdf* (cumulative distribution function). Therefore, the *cdf* for the reliability is $F(r) = r^n$. Taking the derivative of the *cdf*, we get the *pdf* (probability density function) for $r$:

$$f(r) = nr^{n-1} \tag{11}$$

Equation (11) is the distribution of the reliability of a system/component with zero failure during testing.

Bayesian method is often used to estimate reliability when there are few or no failures. When Bayesian theory is applied and no prior information is available for the reliability, a non-informative prior is usually used. There are several commonly used non-informative prior distributions for $r$ [8]. If it is assumed that the reliability is uniformly distributed within [0, 1], the uniform distribution $U(0, 1)$ can be used. This prior is used in [7]. It is the same as a beta distribution $\beta(r;1,1)$. A beta distribution is defined as:

$$\beta(r;a,b)=\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}r^{a-1}(1-r)^{b-1} \tag{12}$$

where $0 \le r \le 1$, $a,b>0$. $\Gamma(\cdot)$ is the Gamma function.

Another non-informative prior, $\beta(r;0.5,0.5)$, is also recommended in [9]. Sometimes, the so called Jeffreys' rule is used to get the non-informative prior [16]. It states that "In general, an approximate non-informative prior is taken proportional to the square root of Fisher's information." Applying this rule, if we use $1/r$ as the prior for $r$, the posterior distribution for $r$ is:

$$f(r \mid data)=\frac{r^n \frac{1}{r}}{\int_0^1 r^n \frac{1}{r}dr}=\frac{r^{n-1}}{\int_0^1 r^{n-1}dr}=nr^{n-1} \tag{13}$$

Equation (13) yields the same *pdf* as the one obtained from equation (11). Therefore, we have proved that the Clopper-Pearson method can be treated as a special case of the Bayesian method. Since equation (11) is found from the exact formula given by the Clopper-Pearson method, no assumption on $r$ is required. So the *pdf* in equation (11) and (13) is the exact distribution for $r$ when there are no failures in the tests. It also can be seen that this distribution is in fact a beta distribution $\beta(r;n,1)$.

Using the above obtained *pdf*, the mean and the variance of the reliability for components with zero failures can be calculated as:

$$E(r)=\frac{n}{n+1}; \tag{14}$$

$$Var(r)=\frac{n}{(n+1)^2(n+2)}; \tag{15}$$

For the general cases when the number of failures in a test is not zero, in the Appendix we prove that the reliability still follows a beta distribution. The *pdf* for reliability $r$ is:

$$f(r)=\frac{\Gamma(n+1)}{\Gamma(n-y)\Gamma(y+1)}r^{(n-y)-1}(1-r)^{(y+1)-1} \tag{16}$$

where $n$ is the sample size and $y$ is the number of failures. Equation (16) is can also be obtained by using $1/r$ as the prior for $r$ through the Bayesian formula similar to equation (13). The proof is not given due to space constraints.

## *4 SYSTEM RELIABIITY AND ITS CONFIDENCE INTERVALS*

Once the distribution of the component reliability is obtained from the method in section 3, the system reliability distribution can now be estimated. Since the system reliability is a function of the component reliabilities, it becomes an issue of obtaining the distribution of a function of random variables. However, obtaining the exact distribution of the system reliability is not always an easy task. In this section, we propose a straightforward procedure to calculate the mean, the variance and the confidence interval of the system reliability. The proposed procedure for estimating the system reliability with zero failure component data involves 4 steps:

1. Calculate the mean and the variance for the reliability of each component using its *pdf*.
2. Calculate the mean and the variance for the system reliability using the means and variances of component reliabilities.
3. Approximate the system reliability distribution using a beta distribution with the calculated mean and variance of system reliability.
4. Compute the CI for the system reliability using the approximated distribution.

### *4.1 Mean and Variance for Component Reliability*

From section 3, it is known that the reliability for each component with zero failure follows a beta distribution with $\beta(r_i;n_i,1)$. Therefore, equations (14) and (15) are used to estimate the component reliability and its variance.

### *4.2 Mean and Variance for System Reliability*

*1) Series System*

The reliability for a series system is calculated by:

$$r_s=\prod_{i=1}^{k} r_i \tag{17}$$

Since $r_i$ follows the beta distribution, $r_s$ is the product of multiple independent beta random variables. The exact distribution for $r_s$ can be found using the Mellin transform [17]. However, this calculation is very computational intensive. To simplify the calculation, Thompson and Haynes [18] suggested approximating the exact distribution with a beta distribution having the same first two moments. In fact, without getting the exact or approximated distribution for $r_s$, its mean and variance can be calculated by:

$$E(r_s)=\prod_{i=1}^{k}E(r_i)=\prod_{i=1}^{k}\frac{n_i}{n_i+1} \tag{18}$$

$$Var(r_s)=E(r_s)\prod_{i=1}^{k}\frac{n_i+1}{n_i+2}-E^2(r_s) \tag{19}$$

This is because for a random variable $X$, $X=\prod_{i=1}^{k}X_i$, if $X_i \sim \beta(x_i;a_i,b_i)$, the mean and the variance for $X$ are [19]:

$$E(X)=\prod E(X_i)=\prod\frac{a_i}{a_i+b_i} \tag{20}$$

$$Var(X)=E(X)\prod\frac{a_i+1}{a_i+b_i+1}-E^2(X) \tag{21}$$

*2) Parallel System*

The reliability for a parallel system is calculated by;

$$r_s=1-\prod_{i=1}^{k}(1-r_i)=1-\prod_{i=1}^{k}q_i \tag{22}$$

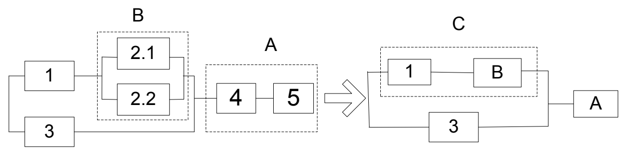
Since $r_i \sim \beta(r_i;n_i,1)$ and $q_i=1-r_i$. $q_i$ is also a beta distribution $q_i \sim \beta(r_i;1,n_i)$. Formulas similar to equation (18) and (19) can be used to get the mean and variance for $r_s$ for a parallel system. They are:

$$E(r_s) = 1 - \prod_{i=1}^{k} E(q_i) = 1 - \prod_{i=1}^{k} \frac{1}{n_i + 1} \quad (23)$$

$$Var(r_s) = [1 - E(r_s)] \left[ \prod_{i=1}^{k} \frac{2}{n_i + 2} - 1 + E(r_s)) \right] \quad (24)$$

*3) Complex System*

A complex system is a system that can be represented by series and parallel subsystems. Let's use an example to explain this. The system given in Figure 1 (a) can be simplified to the system in Figure 1 (b) by combining some components.




(a) (b)

*Figure 1 – System with Complex Configuration*

Assuming that each component in Figure 1 (a) had no failures during the component tests, the reliability of each component follows $\beta(r_i; n_i, 1)$. The parallel subsystem consisting of 2.1 and 2.2 in Figure 1 (a) can be represented by block B in Figure 1 (b). The mean $E(r_B)$ and variance $Var(r_B)$ of the reliability of block B can be calculated using equations (23) and (24). Similarly, the mean and variance of the reliability of subsystem A (components 4 and 5) can be estimated using equations (18) and (19). If we keep combining subsystems in such manner, the system finally can be represented by a single block.

Unfortunately, the exact distribution for block B is unknown. We only know that its reliability is a product of two beta random variables. So the variance for the subsystem consisting of component 1 and block B cannot be calculated using equation (19), which is for the product of two independent beta random variables. However, the biased variance can be estimated using the formulas below.

If $X = \prod_{i=1}^{k} X_i$ and the mean and the variance for $X_i$ are known, the variance for $X$ can be estimated by [2]:

$$Var(X) = \prod_{i=1}^{k} \left[ E^2(X_i) + Var(X_i) \right] - \prod_{i=1}^{k} E^2(X_i) \quad (25)$$

Equation (25) can be used to calculate the variance for the series system consisting of 1 and B.

Similarly, there is a formula for parallel blocks. If $X = 1 - \prod_{i=1}^{k} (1 - X_i)$, the variance for $X$ can be estimated by:

$$Var(X) = \prod_{i=1}^{k} \left[ E^2(1 - X_i) + Var(X_i) \right] - \prod_{i=1}^{k} E^2(1 - X_i) \quad (26)$$

Equation (26) can be used to calculate the variance for the parallel system consisting of C and 3.

By keeping calculating the reliability variance for the combined subsystems using equation (25) and (26), the variance of the final system reliability can be obtained. The mean value of the system reliability is calculated by:

$$E(r_s) = g(E(r_1), E(r_2).,..., E(r_k)) \quad (27)$$

where $r_s = g(r_1, r_2, ..., r_k)$ is the function for getting the system reliability $r_s$ from its component reliability $r_i$.

*4.3 Calculate the CI for System Reliability*

Once the $E(r_s)$ and $Var(r_s)$ are obtained, we will approximate the system reliability distribution by a beta distribution $\beta(r_s; a, b)$. For a random variable *X* following a beta distribution with parameters *a* and *b*, its mean and variance are:

$$E(X) = \frac{a}{a + b} \quad (28)$$

$$Var(X) = \frac{ab}{(a + b)^2 (a + b + 1)} \quad (29)$$

Setting equations (28) and (29) to the estimated mean and variance obtained from the procedure in section 4.2, parameters *a* and *b* can be estimated using:

$$a = E(\hat{r}_s) \left( \frac{E(\hat{r}_s) - E^2(\hat{r}_s)}{Var(\hat{r}_s)} - 1 \right) \quad (30)$$

$$b = \left(1 - E(\hat{r}_s)\right) \left( \frac{E(\hat{r}_s) - E^2(\hat{r}_s)}{Var(\hat{r}_s)} - 1 \right) \quad (31)$$

Once *a* and *b* are found, the one-sided 100*CL*% lower bound for the system reliability $r_{sL}$ can be calculated by:

$$\int_0^{r_{sL}} \beta(r_s; a, b) dr_s = 1 - CL \quad (32)$$

By now we have obtained the approximated beta distribution for the system reliability using the component test data. Is this approximation accurate? Simulations for systems with different configurations have been conducted to evaluate the accuracy of the approximation. It was found that the beta distribution approximation is accurate enough. If, for some reason this approximation is not accurate, bootstrap confidence bounds can always be used for the system reliability.

Another reason of using beta distribution for the approximation is because it can be easily incorporated in the reliability demonstration test design due to its conjugate property. In Bayesian theory, if a conjugate distribution is used as the prior distribution, the resulting posterior distribution will be in the same distribution family. $\beta(r_s; a, b)$ is the prior information for the system reliability and should be used in the system test design.

## *5 RELIABILITY DEMONSTRATION TEST DESIGN*

If there is no prior knowledge on the system reliability, equation (8) is used to design the test plan for the reliability demonstration. In equation (8), there are four unknown parameters: $n$ the sample size; $y$ the number of allowed failures, $r_L$ the demonstrated lower reliability and *CL* the confidence level. Knowing any 3 of them, the remaining one can be found.

However, equation (8) ignores the system reliability information from the component test data. The information on system reliability estimated from section 4 can and should be used in test planning.

For example, to demonstrate a system reliability of 0.95 at a 95% confidence level, by using prior knowledge the number of samples can be solved by using the following equation:

$$\int_0^{r_{sL}} \beta(r_s; a+n, y+b) dr_s = 1 - CL \quad (33)$$

where $a$, $b$ are obtained from the component test data and $\beta(r_s; a, b)$ is the prior distribution for the system reliability; $y$ is the allowable number of failures in the final system test; and $n$ is the sample size that must be determined. We can see that in equation (33) the posterior distribution for $r_s$ is a beta distribution $\beta(r_s; a+n, y+b)$ .

Since the reliability information from component tests is included in equation (33), a more efficient test plan is expected when equation (33) is used to calculate the sample size. An example is given in section 6 to illustrate this.

## 6 EXAMPLE

For the example in Figure 1, the subsystem test data is summarized in Table 1.

| Component | Units Tested | Failures |
|---|---|---|
| 1 | 100 | 0 |
| 2 | 120 | 0 |
| 3 | 180 | 0 |
| 4 | 50 | 0 |
| 5 | 70 | 0 |

*Table 1 – Subsystem Test Data*

If a subsystem is not a one-shot system, the above failure data represents the on-demand or mission failures. Components 2.1 and 2.2 are the same type of components but they are functioning independently. From Table 1 and section 3, we know the reliability of each component follows $\beta(r_i; n_i, 1)$, where $n_i$ is the number of tested units for each component. Following the procedure in section 4, the system configuration can be further simplified from Figure 1(b) to:

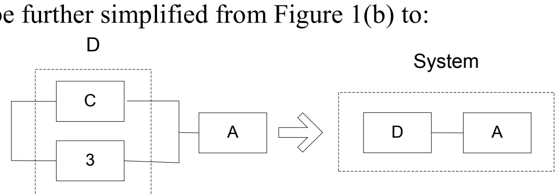



*Figure 2 – Simplified System*

Using the method in section 4, the calculated mean and the variance for the reliability at each step are given in Table 2.

| Block | Mean | Variance |
|---|---|---|
| A | 0.96658 | 5.448E-04 |
| B | 0.99993 | 1.369E-08 |
| C | 0.99003 | 9.611E-05 |
| D | 0.99994 | 8.835E-09 |
| System | 0.96653 | 5.447E-04 |

*Table 2 – Estimated Mean and Variance at Each Step*

Using the estimate mean and variance in Table 2, the system reliability distribution is approximated by a beta distribution $\beta(r_s; 56.43127, 1.954125)$. The one-sided 95% lower bound for the system reliability using different methods are compared in Table 3.

| Method | Description | 95% Lower Bound |
|---|---|---|
| 1 | Bayesian with beta(0.5, 0.5) as the Prior [7, 9] | 0.94969 |
| 2 | Bayesian with beta(1, 1) as the Prior [6, 8] | 0.92255 |
| 3 | Assume *Rs* ~ Lognormal Distr. [2] | 0.92917 |
| 4 | The Proposed Integrated Method in this Paper | 0.92122 |
| 5 | Simulation | 0.92121 |

*Table 3 – Results from Different Methods*

The simulation in Table 3 is conducted using Matlab with a seed of 0 and number of runs of 5,000. The exact distribution of the reliability in equation (11) is used to generate the random numbers for each component. 5,000 values of the system reliability are obtained and sorted in an ascending order. The 250$^{th}$ ($5{,}000 \times 0.05$) value is the one-sided 95% lower bound for the system reliability. From Table 3, we can see that using $\beta(r_i; 0.5, 0.5)$ as the non-informative prior is not appropriate. Method 3 in Table 3 cannot be directly used for the zero failure tests. In order to use it, we use the proposed procedure in section 4 to get the variance and mean values for the system reliability. Then the system reliability is assumed to be lognormally distributed to get the lower bound, as in [2]. Table 3 shows that for this example, the lognormal distribution assumption is not as accurate as the beta distribution approximation. Figure 3 is the *cdf* of the approximated beta distribution with the simulation data.

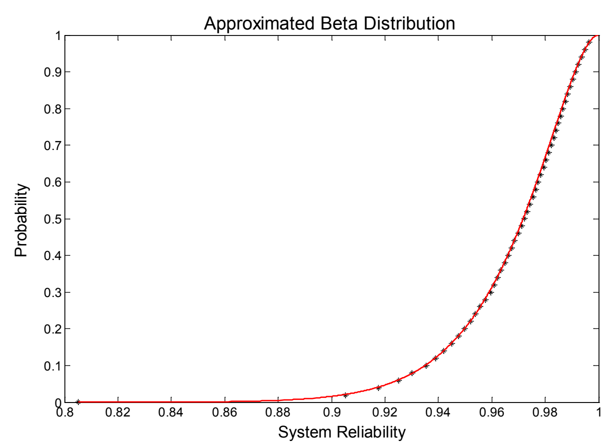



*Figure 3– Approximated beta (a,b) vs. Simulation Data*

The curve is the *cdf* from $\beta(r_s; 56.43127, 1.954125)$. The star points are the estimated probability using the median ranks for the simulation data. From Figure 3, it can be seen that the beta distribution matches the simulation data very well. Please notice that $\beta(r_s; 56.43127, 1.954125)$ is not estimated from the simulation data, rather $a$ and $b$ are calculated from equation (30) and (31).

With the prior information on the system reliability, a better test plan can be designed. Assume that we want to design a zero failure test to demonstrate the system reliability of 0.95 at a 95% confidence level. If equation (9) is used directly without using the system reliability information estimated from its component test data, the required sample size for the test is 59. If we use $\beta(r_s; 56.43127, 1.954125)$, the information estimated from the component tests, the sample size calculated using equation (33) is 35.

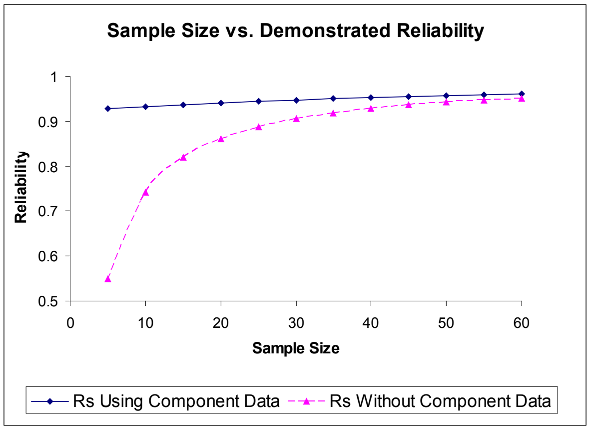



*Figure 4– Comparison of Test Plans*

Figure 4 shows the relationship of the sample size and demonstrated system reliability at a confidence level of 95%. The dotted curve is test plans without using the component data. It can be seen that the difference between using and not using the component test data is significant when the system test sample size is small.

## 7 CONCLUSIONS

In this paper, a method of estimating system reliability and its confidence bounds from zero failure component tests was proposed. This proposed method integrates the advantages of the existing methods and is very easy to use. It also can be extended to the general cases where failures occur in component tests. We also mathematically proved that the Clopper-Pearson or the beta binomial method is a special case of the Bayesian method. Since the Clopper-Pearson method uses the binominal equation directly, no assumption is required for the distribution of the reliability. When it is used together with the Bayesian method, the results are more accurate. An example was used to illustrate the accuracy of the proposed method. The example also showed that using the information from the component tests can efficiently reduce the sample size in the final system reliability demonstration test.

## 8 APPENDIX

From the equation of Clopper-Pearson method:

$$1-CL=\sum_{k=0}^{y}\binom{n}{k}(1-r_L)^k r_L^{\,n-k}$$

We get:

$$F(r)=\sum_{k=0}^{y}\binom{n}{k}r^{n-k}(1-r)^k$$

Take the derivative to $r$ to get the *pdf* $f(r)$:

$$f(r)=-\sum_{k=0}^{y}\binom{n}{k}k(1-r)^{k-1}r^{n-k}+\sum_{k=0}^{y}\binom{n}{k}(1-r)^k r^{n-k-1}(n-k)$$

$$=\sum_{k=0}^{y}\frac{n!}{k!(n-k)!}(1-r)^k r^{n-k-1}(n-k)$$

$$-\sum_{k=0}^{y}\frac{n!}{k!(n-k)!}k(1-r)^{k-1}r^{n-k}$$

$$=\sum_{k=0}^{y}\frac{n!}{k!(n-k-1)!}(1-r)^k r^{n-k-1}$$

$$-\sum_{k=1}^{y}\frac{n!}{(k-1)!(n-k)!}(1-r)^{k-1}r^{n-k}$$

When $k$ = 0, the second term is 0. So we can start from $k$ = 1. Let $k'=k+1$ in the second term, then

$$f(r)=\sum_{k=0}^{y}\frac{n!}{k!(n-k-1)!}(1-r)^k r^{n-k-1}$$

$$-\sum_{k'=0}^{y-1}\frac{n!}{k'!(n-k'-1)!}(1-r)^{k'}r^{n-k'-1}$$

In the above equation, only the $k=y$ term is left. The remaining terms cancel. So:

$$f(r)=\frac{n!}{y!(n-y-1)!}r^{(n-y)-1}(1-r)^{(y+1)-1}$$

This is the *pdf* for a beta distribution $\beta(r;n-y,y+1)$.

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