Reliability Evaluation and Application for Systems with Different Phases

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Key Words: Accelerated life testing, cumulative damage, non-repairable systems, phased-mission systems, repairable systems

SUMMARY & CONCLUSIONS

Many systems are multi-phase mission systems. During different phases, the system reliability configuration, reliability requirements and operation environment can be different. For example, some components may change from active status to standby or the system reliability configuration may change from serial to parallel. In addition to configuration changes, the system may be exposed to different stress conditions; therefore, the time to failure distribution of some components will change due to the change in stress. In general, systems that operate under different phases are considered as dynamic systems. A way to estimate the reliability and availability of such systems is needed.

In this paper, a systematic procedure of applying accelerated life tests and simulation to analyze the reliability and availability of such dynamic systems is first proposed. Methods for solving both non-repairable and repairable systems are provided. For non-repairable systems, an analytical solution based on cumulative damage theory is discussed. Therefore, the exponential assumption, which is used in many existing methods, is not required in the proposed method. In addition to the analytical solution, a cumulative-damage-based simulation solution is also provided. For repairable systems, based on different scenarios in real applications, the repairable phased-mission system is classified into three categories. Because of the complexity of the problem, only simulation results are given for repairable systems.

The proposed systematic procedure of applying accelerated life tests in phased-mission system analysis provides a general guideline for dealing with real-world applications. The cumulative-damage-based analytical and simulation method provide a practical and useful approach for solving phased-mission system problems. The classification of repairable phased-mission systems gives a clear overview of the phased-mission systems and makes the analysis of complex real applications easier.

1 INTRODUCTION

Phased-mission systems are often encountered in many practical applications. A classic example is a twin-engine airplane, which involves taxiing, take-off, cruising and landing phases during each trip. In the taxi phase, one engine is required, while in the take-off phase, two engines are necessary. During each phase, the stresses applied on the engines, such as load, vibration, etc., are different; therefore, different life distributions should be used to evaluate the engine reliability and system availability at different phases.

Phased-mission systems have been studied for more than three decades. If the system is non-repairable and only one mission cycle is considered, the binary decision diagram (BDD) method can be applied [1]. However, the BDD method is limited by the problem size. When the system reliability configuration is complicated and the number of components is large, especially when the system reliability of multiple mission cycles needs to be evaluated, using a BDD will not be easy and straightforward. If the system is repairable and the life and repair time distributions of components are exponential, the continuous time Markov chain methods [2] are well developed. However, in most cases the failure and repair distributions are not exponential, and a more general approach is needed.

The challenge in developing an analysis method for phased-mission systems is that there could be a sharp change in system reliability due to the change of the system RBD (reliability block diagram) or change of the failure characteristics of the components. Due to this complexity of phased-mission systems, analytical solutions are available only for some special and simple cases [1, 2]. So far, no good method exists for solving general cases of phased-mission systems, which are more often encountered in actual applications. With the advance in computer technology, computer speed is becoming faster, which makes the alternative method of simulation a very viable approach for analyzing such dynamic systems [3].

In this paper, we will illustrate how to use simulation to study phased-mission systems. A cumulative-damage-based simulation method is proposed. To help the analysis, we classify the phased-mission systems into two major classes: repairable systems and non-repairable systems. According to the different strategies that the system may use to respond to a failure, the repairable systems are further classified into three different categories.

- Type I repairable system – End Mission,
- Type II repairable system - Continue Mission,
- Type III repairable system - Go to Maintenance.

Details of these definitions will be given in Section 4. In summary, the phased-mission systems are classified as shown in Figure 1.

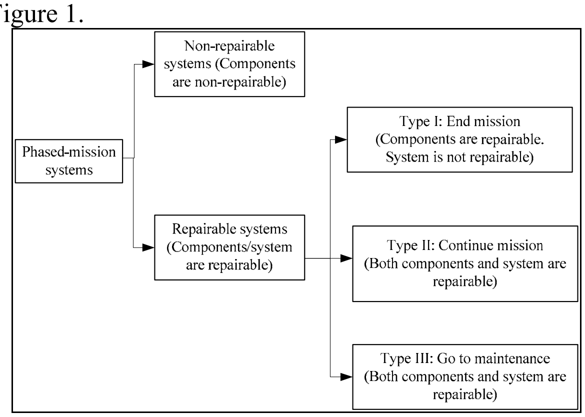


Figure 1 – Classification of Phased-Mission Systems

The paper is organized as follows. In Section 2, an example of a phased-mission system is given. This example will be used through the paper. An accelerated life test and analysis will then be used to illustrate how to obtain the life distribution of each component at each phase. Section 3 discusses the analytical reliability solution and the simulation solution for the non-repairable cases. In Section 4, the details of using simulation to study the availability of repairable systems are given. Conclusions are discussed in Section 5.

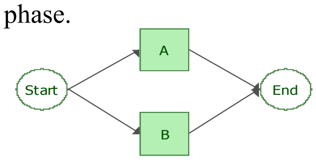
2 PROBLEM DEFINITION AND FAILURE DISTRIBTUION DETERMINATION

In this section, a general example will be defined. Specifically, it is a dynamic system whose RBD and environmental stresses change at each phase. The procedure of solving this particular problem that will be described can easily be extended to other phased-mission systems.

2.1 *An Example of Phased-Mission System*

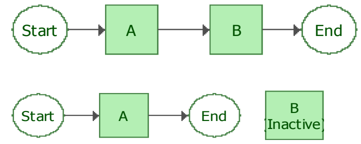
Consider a system that has three different phases within a mission cycle. Multiple repeated mission cycles can be conducted by the same system. The structural configuration of each phase is given in Figure 2.

In phase 3, component B is inactive and therefore is not aging. To increase the complexity of the problem, let's assume that this system experiences a different load at different phases. The load (stress) profile of each phase is given in Figure 3. Each profile is applied to the entire phase duration of the corresponding phase.



Phase 1:

Phase 2:



Phase 3:

Figure 2 – Example: RBDs of a System with 3 Phases

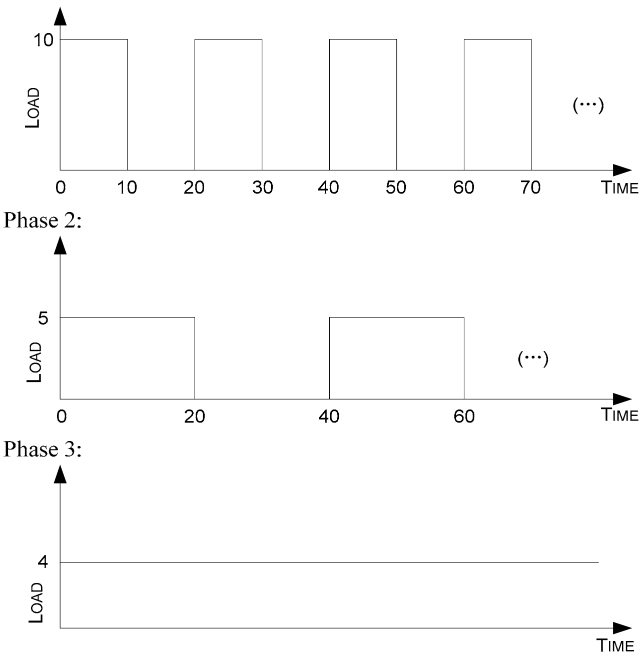


Figure 3 – Example: Stress Profile of a System with 3 Phases

2.2 *Determination of Life Distribution Using Accelerated Life Tests*

From the stress profiles in Figure 3, it can be seen that there are two types of stresses. One is the load and the other is the frequency. In phase 1, the load level is 10 and the frequency is 50 cycles per 1,000 time units. In phase 2, the load level is 5 and the frequency is 25 cycles. In phase 3, the load level is 4, applied constantly. All the stresses are applied to both components A and B in each phase.

In order to estimate the lifetime distribution of each component at each phase, an accelerated life test was first conducted. The test plan for component A is given in Table 1.

Sample Size	Load	Frequency
20	20	50
30	10	100
20	5	200
10	20	200

Table 1 – Accelerated Life Test Plan for Component A

All samples were tested to failure. From the obtained failure data, we can get the probability plot shown in Figure 4.

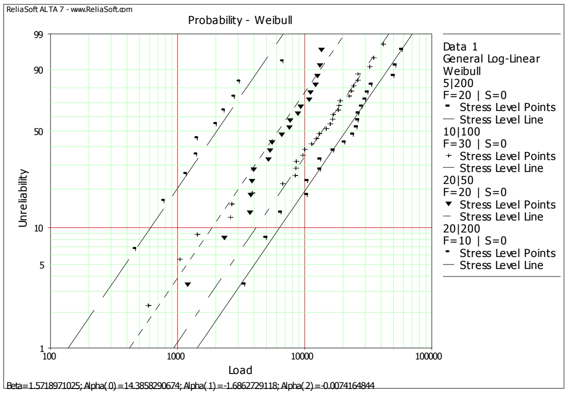


Figure 4 – Weibull Probability Plot for Component A

In Figure 4, the four lines are the probability plot for the four stress conditions given in Table 1. If Weibull distribution is used for component A, the reliability of component A is:

$$R_A(t) = e^{-(t/\eta(S))^{\beta}} \quad (1)$$

The life-stress relationship $\eta(S)$ is:

$$\ln(\eta) = 14.39 - 1.686\ln(S_1) - 0.0074 S_2 \quad (2)$$

where S_1 is the load and S_2 is the frequency. The relationship in (2) also can be illustrated using the life-stress relationship plot in Figure 5.

Figure 5 – Life-Stress Relationship Plot for Component A (Varying Load, Fixed Frequency = 250)

Using equation (1) and (2), we can easily get the lifetime distribution for component A at each phase. A similar method can be used to get the lifetime distribution for component B. The distributions obtained for each component at different phases are summarized in Table 2. Each phase has a duration length of 1,000 time units.

Phase	Stress	Distribution of Component A	Distribution of Component B
Phase 1 ($T1$=1,000)	$S_1 = 10$ $S_2 = 50$	$\beta_A = 1.57$ $\eta_{A,1} = 25{,}282$	$\beta_B = 1.2$ $\eta_{B,1} = 20{,}000$
Phase 2 ($T2$=1,000)	$S_1 = 5$ $S_2 = 25$	$\beta_A = 1.57$ $\eta_{A,2} = 97{,}879$	$\beta_B = 1.2$ $\eta_{B,2} = 50{,}000$
Phase 3 ($T3$=1,000)	$S_1 = 4$ $S_2 = 0$	$\beta_A = 1.57$ $\eta_{A,3} = 171{,}563$	N/A

Table 2 – Lifetime Distributions of Components A and B

3 NON-REPAIRABLE PHASED-MISSION SYSTEMS

In this section, we will derive the analytical solution of the system reliability using the distributions in Table 2 for the example in Figure 2. Unlike the exponential distribution, which has the "memoryless" property (no cumulative damage), for a general Weibull distribution the cumulative damage during each operational phase should be considered in the reliability calculation.

3.1 *Analytical Solution for Non-repairable Phased-Mission Systems*

In order to understand the process of the analytical solution, we will introduce the "Equivalent Age" concept [4]. At the end of phase 1, the reliability for component A is:

$$R_A(T_1) = \exp[-(T_1/\eta_{A,1})^{\beta_A}] \quad (3)$$

This is the reliability of component A at the beginning of phase 2, which reflects the damage accumulated by the end of phase 1. Therefore:

$$R_A(T_1) = R_A(T'_{A,1}) = \exp[-(T'_1/\eta_{A,2})^{\beta_A}] \quad (4)$$

where $T'_{A,1}$ is the "equivalent age" or "age" for short of component A at the beginning of phase 2. Its value is:

$$T'_{A,1} = (\eta_{A,2} T_1)/\eta_{A,1}$$

If the distribution type is different in phase 1 and phase 2, the procedure given in equation (3) and (4) can still be used to get the equivalent age.

During each phase, the reliability for each component will be the conditional reliability based on the equivalent age at the beginning of that phase. Define the end time of the ith phase as T_i^- and the starting time of the (i+1)th phase as T_i^+. At time t within the (i+1)th phase, the system reliability is:

$$R_S(t) = R_S(T_i^-) \times P(\text{System is working at } T_i^+ \mid \text{System is working at } T_i^-) \times P(\text{System is working at } t \mid \text{System is working at } T_i^+) \quad (5)$$

The system reliability for the first mission cycle for this example will be:

$$R_S(t) = \begin{cases} R_A(t) + R_B(t) - R_A(t)R_B(t); & t \le T_1 \\ R_S(T_1^-) \times \left[\dfrac{R_A(T'_{A,1})R_B(T'_{B,1})}{R_A(T'_{A,1}) + R_B(T'_{B,1}) - R_A(T'_{A,1})R_B(T'_{B,1})} \right] \times \dfrac{R_A(t - T_1 + T'_{A,1})R_B(t - T_1 + T'_{B,1})}{R_A(T'_{A,1})R_B(T'_{B,1})}; & T_1 \le t \le T_2 \\ R_S(T_2^-) \times 1 \times R_A(t - T_2 + T'_{A,2})/R_A(T'_{A,2}); & T_2 \le t \le T_3 \end{cases} \quad (6)$$

where $R_S(t)$ is the system reliability at time t; $R_S(T_i^-)$ is the reliability at the end of phase i; $T'_{A,i}$ is the age of component A at the beginning of phase i+1 (the end of phase i). In this particular example, and for the first mission cycle of phase 2, equation (6) can be reduced to:

$$R_A(t - T_1 + T'_{A,1})R_B(t - T_1 + T'_{B,1}); \quad T_1 \le t \le T_2 \quad (7)$$

The analytical solution for 4 consecutive mission cycles for the example is show in Figure 6.

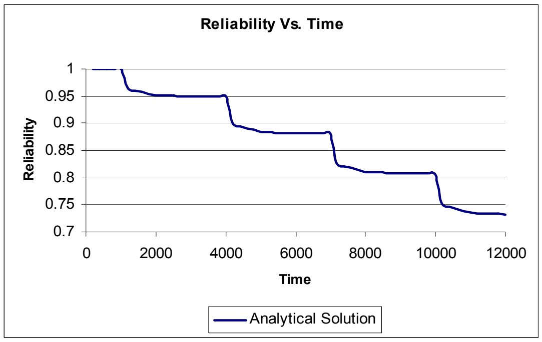


Figure 6 – Analytical Solution for a Three Phase System

3.2 *Further Discussion of Analytical Solution of Phased-Mission Systems*

For the example discussed above, we can extend equation (5) to get the reliability of the system with more than one mission cycle. The probability that the system is working at the beginning of phase 1, given that it is working at the end of phase 3, is 1. For the following cycles, the probability that the system is working at the beginning of phase 2, given that it is working at the end of phase 1, is still:

$$R_A(T_{A,1}^-)R_B(T_{B,1}^-)/R_S(T_1^-)$$

Therefore, the reliability during phase 2 for any cycle can still be calculated using equation (7), where T_1 is the end time of phase 1. However, depending on the system configuration, even for a relatively simple system, it is not an easy task to get the reliability equation. For example, for a system in Figure 7, it is not an easy task to get the reliability equation.

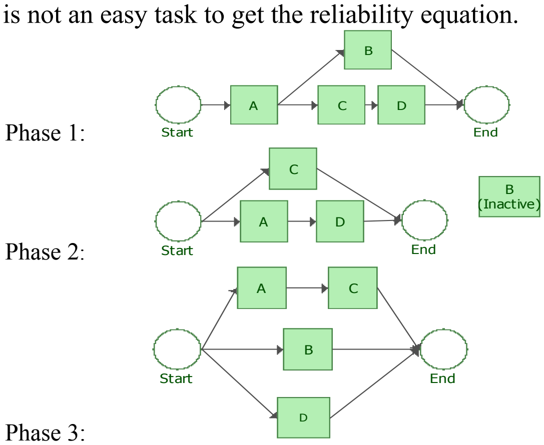


Figure 7 – A Relatively Complex Phased-Mission System

Other methods that can be used to obtain reliability for simple non-repairable systems were discussed in [1, 5]. However, with increasing number of components and mission cycles, it is really a challenging task for all the existing methods.

3.3 *Simulation Solution for Non-repairable Phased-Mission Systems*

From the above discussion, it can be seen that obtaining analytical solutions, even for a simple phased-mission system, is not easy. For problems that become analytically intractable, simulation can be used as an alternative tool. The simulation method can handle very complex systems within acceptable accuracy and speed. For the example discussed before, the simulation results are plotted along with the analytical solution in Figure 8.

To get the simulation results in Figure 8, 3,000 simulation runs with the simulation end time of 12,000 were conducted. It took about 10 seconds on a PC with CPU speed of 2.80GHz and RAM of 510MB. The detail of the cumulative-damage-based simulation method used to get the results is given in Section 4.

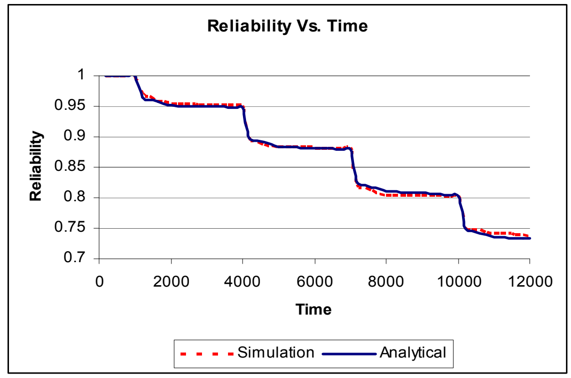


Figure 8 – Simulation and Analytical Solution for a Three Phase System

As we know, from the reliability equation (if it can be found), the importance of each component can be obtained by taking the derivative respective to each individual component. Simulation can also be utilized to identify which components are critical to the system reliability. We define an index called the *Failure Criticality Index* (FCI) to quantify the reliability importance of a component. The FCI for component A is calculated by:

$$\frac{\textit{Number of system failures caused by A by time t}}{\textit{Total number of system failures by time t}}$$

The FCI comparison of component A and B is given in Figure 9.

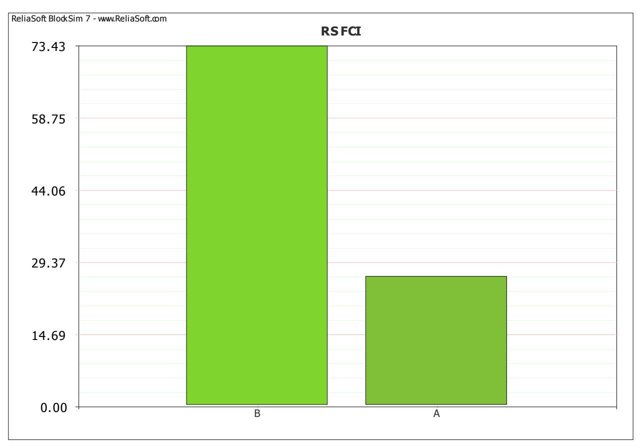


Figure 9 – Failure Critical Index of Components A and B

From the plot, it can be seen that component B caused about 73% of the system failures. This FCI can also be obtained for a given phase to determine the most critical components in that particular phase.

4 REPAIRABLE PHASED-MISSION SYSTEMS

In this section, we classify all phased-mission repairable systems into three categories. These three classes cover a wide range of real-world applications and provide a clear overview of the repairable phased-mission systems.

- *Type I Repairable Phased-Mission System: End Mission*

For this type of system, the component is repairable if the system continues to operate after the component fails. However, if there is a system failure, no repair is conducted. The system is lost and the mission ends. Consider the aircraft mentioned previously. During the cruising phase, catastrophic failure in the system is non-repairable and aborts the mission. However at the component level, the components may be repairable (most likely during a different phase) if there is no system failure.

- *Type II Repairable Phased-Mission System: Continue Mission*

For this type of system, all the components are repairable even when the system has failed. For example, consider a manufacturing line operating in three shifts. Given that the production line operates at different capacities during different shifts, components are used differently (or not used) during these shifts. Each shift can be treated as a phase. Assume that during shift 2, there is a system failure and repair is conducted to fix the failure. The repairs continue beyond the 2nd shift and are completed sometime during the 3rd shift. Once the failed components are repaired, they will operate at the capacity needed for the 3rd shift.

- *Type III Repairable Phased-Mission System: Go to Maintenance*

For this type of system, one or more maintenance phases are scheduled at pre-determined times. When a system failure occurs during an operational phase, the system will go to the maintenance phase directly, skipping any other operational phases that might exist between the current operational phase and the specified maintenance phase. In the maintenance phase, all the failed components will be fixed. For the components that did not fail, preventive maintenance will be conducted if they meet pre-defined preventive maintenance criteria. For example, consider a military vehicle with three operational phases. At the end of each mission cycle (all three phases completed), the vehicle is sent for maintenance for any required actions. After finishing this scheduled maintenance, the vehicle will start a new mission cycle. However, if there is a system failure during any one of the operational phases, the vehicle will be sent to maintenance without completing the mission cycle. The phase diagram for this example is shown in Figure 10.

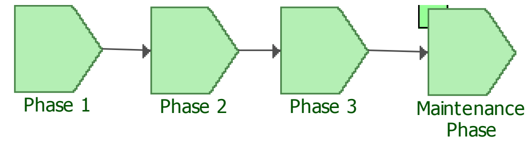


Figure 10 – Example of a Type III Repairable System

In order to consider the cumulative damage accrued from previous phases in the simulation, the "age" and the corresponding failure distribution are recorded for each component. Equations (3) and (4) are used to transfer the ages between different phases. For instance, if component A fails in phase 1, say at time t_1 ($t_1 < 1{,}000$), and was repaired at time t_2, several methods can be used to get the age at t_2 based on different assumptions of the repair effect. If the repair is a *minimal repair*, in which the repair restores A to the status that it had when it failed, A will have the same reliability as what it had at time t_1. Using this reliability and the failure distribution at time t_2, the corresponding "equivalent age" of A at time t_2 can be calculated using (4). If the repair is a *perfect repair* and component A is restored to a brand-new status, the "age" will be 0 and the reliability is 1 at time t_2. It means that all of the accumulated damage is removed by the repair. If the repair restores component A to a status where its reliability is higher than what it had at time t_1 but is lower than 1, this type of repair is called *general repair*. In this paper, we assume the repair is minimal repair.

By adjusting the "age" of a failed component after it is repaired, simulation can easily handle different assumptions on the repairs and make it more flexible and more realistic than analytical methods. For more discussions on the repair types, please refer to [6, 7].

In the following sections, we will study the three different types of repairable systems discussed at the beginning of this section. The same example given in Figure 2 will again be used. The repair time distributions (normal distribution) are given in Table 3 below.

Type	Phase	A	B
Type I & II System	Phase 1	$N(200,20^2)$	$N(500,40^2)$
	Phase 2	$N(400,40^2)$	$N(1000,50^2)$
	Phase 3	$N(800,10^2)$	N/A
Type III System	Maintenance Phase	$N(1000,80^2)$	$N(2000,200^2)$

Table 3- Repair Time Distribution of Each Component

4.1 *Simulation Solution for Type I Repairable Phased-Mission Systems (End the Mission)*

In Type I situations, upon system failure, the system will remain down until the end of the simulation time and then start a new simulation run. Using the distribution settings in Table 2 and 3, we can get the availability of the system below.

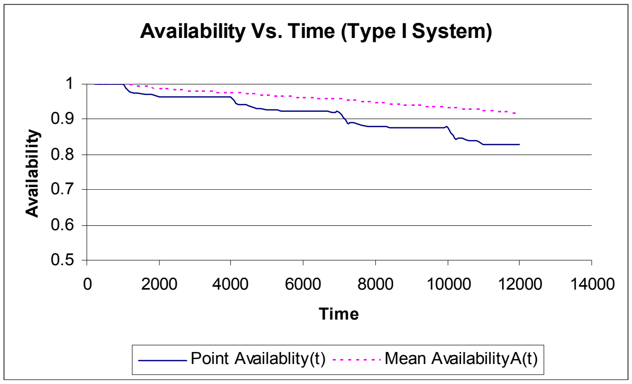


Figure 11 –Simulation Result for Type I Systems

The results in Figure 11 were obtained by conducting 3,000 runs with the simulation end time of 12,000.

In Figure 11, point availability at time *t* is used to measure the probability that a system is working at time *t*. Mean availability at time *t* is used to measure the portion of time that the system is working from time 0 to time *t*.

4.2 *Simulation Solution for Type II Repairable Phased-Mission Systems (Continue Mission)*

In Type II situations, the system will be repaired immediately upon system failure. Repairs are part of the operational phase's time and might continue into the next phase if not completed by the end of the phase in which the failure(s) occurred. In other words, after the repair is completed, the system will continue its mission from the time at which the system is fixed. This option is the most similar to a traditional RBD with the only exception being that the cumulative damage is transferred from phase to phase to account for changes in failure distributions of components. In a traditional RBD, which also can be treated as a phased-mission system with only one phase, the transfer of age is not necessary because the failure distribution is always the same. Using the settings in Table 2 and Table 3, we can get the availability of the system below.

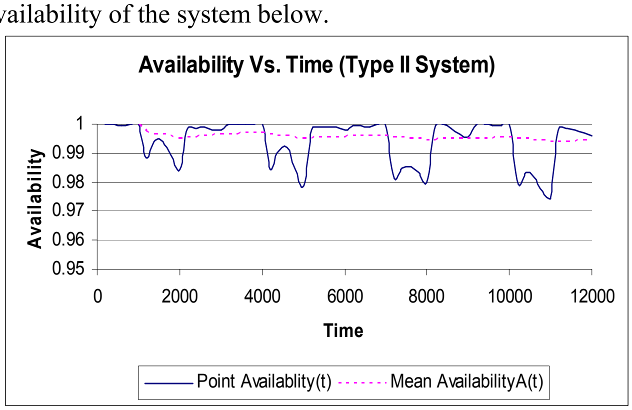


Figure 12–Simulation Result for Type II Systems

4.3 *Simulation Solution for Type III Repairable Phased-Mission Systems (Go to Maintenance)*

In Type III situations, operational phases are suspended immediately when a system failure occurs and the system is sent to maintenance. In this example, when all specified maintenance actions are completed, the system will start a new cycle (since there are no operational phases defined after the maintenance phase). Results for the availability of the system are given in Figure 13.

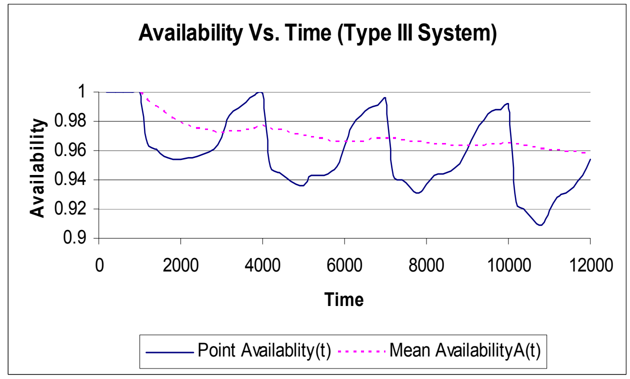


Figure 13–Simulation Result for Type III Systems

As with the results for the non-repairable systems in Section 3, the FCI of repairable systems also can be obtained from the simulation. These results are not shown here.

4.4 *Additional Metrics of Interest*

When dealing with phased-mission systems using simulation, reliability and availability give only a partial picture of the performance of the system. Therefore other metrics must be used to evaluate the capabilities of such systems. Those metrics are discussed in detail in Ref [8]. In a similar manner, overall results for the life of a system (*e.g.* number of system failures or maintenance actions, system downtime, component level results, etc.) give limited information about a system that dynamically changes over time. However, by obtaining such results for specific phases, the analyst can target not only specific critical factors (components, resources, maintenance policies, etc.), but also critical phases and critical factors within such phases.

An additional factor becomes critical on phased-mission systems that have repeated missions over time, or mission cycles, like the aircraft example previously mentioned. It is important to evaluate the system at different phases, such as during cruising or landing, but it is also of great importance to evaluate changes over mission cycles. For example, what is the probability that the aircraft will be able to complete 10 trips or what is the probability that it will be able and ready to start a 5^{th} trip? Metrics such as *EPPA* (end-of-phase point availability), *Mean Duration* of each phase and *Phase Reliability*, which is the probability that the system does not fail during the phase in a given mission cycle, may be useful. For more details, please refer to [8].

4.5 *Further Discussion of Simulation Method for Phased-Mission Systems*

Just like in the analytical solution, the "equivalent age" or "age" is the key in the simulation for non-repairable systems. The "equivalent age" of a component represents the cumulative damage it has accrued up to that point in time.

Using "equivalent age" makes the simulation very flexible and able to handle many real applications such as duty cycles, different repair types. The "equivalent age" can be easily adjusted based on the duty cycle factor or the repair effectiveness factor.

5 CONCLUSIONS

In this paper, we proposed a systematic method to analyze phased-mission systems by using accelerated life tests and simulation. A cumulative-damage-based analytical solution for the non-repairable system example is given. A simulation method that is also based on cumulative damage theory is applied to both non-repairable and repairable phased-mission systems. For the repairable systems, we proposed to classify them into three different categories. This classification provides a great help on analyzing complex systems under different scenarios and reliability requirement. Because of the complexity of the phased-mission systems, especially for repairable systems in real applications, finding analytical solutions becomes very challenging. Therefore, the proposed cumulative-damage-based simulation method is a very useful tool for handling such problems in real applications.

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