# Application of Quantitative Accelerated Life Models on Load Sharing Redundancy

Adamantios Mettas; ReliaSoft Corporation, Tucson
Pantelis Vassiliou; ReliaSoft Corporation, Tucson

Key Words: Load Sharing, System Analysis, Accelerated life testing, Time-varying stresses, Time-dependent stresses, Cumulative damage, Step-stress, Maximum likelihood

## SUMMARY & CONCLUSIONS

In most cases, when analyzing redundancy, independence is assumed across the components within the system. In other words, it is assumed that the failure of a component does not affect the failure distributions of the remaining components. However, if a system consists of multiple components sharing a load then the assumption of independence no longer holds true. If one component fails then the component(s) that are still operating will have to assume the failed unit's load. Therefore, the reliabilities of the surviving unit(s) will change. Calculating the system reliability is no longer an easy proposition. This paper will explore the concept of component dependence while using accelerated life test data analysis models to determine system reliability. The cumulative damage model will be used to analyze life data obtained from load sharing components, and solve for the parameters of the life-stress model. The life-stress model will then be used as an input to the system's reliability equation, therefore providing a comprehensive model for performing inferences on the life of load sharing components based on their operating stress. Utilizing these models provides better understanding of the behavior of the system in the field, and allows for more intelligent predictions.

## 1. INTRODUCTION

Consider the case where the reliability of a system with two components in parallel is to be calculated. In most cases it is assumed that the components are statistically independent in order to simplify the analysis. In real applications, however, if one component fails then the component that is still operating will have to assume the failed unit's load. Therefore, the failure distribution of the surviving unit will change, since it will have to carry 100% of the load. This obviously will have a great impact on the reliability of the surviving component. This will continue for as long as the other component is down. While this is occurring, the probability of the surviving component failing due to the increased load (stress) will also be increased (compared to when the two components share the load equally). This type of arrangement is called "Load Sharing." It is clear that in order to calculate the reliability of such a system, two life distributions need to be defined. The first would be the life distribution of each component when both components operate, and the second would be the life distribution of each component when only one is operating and carrying the entire load. This translates into testing the components under two different loads and obtaining a life distribution at each load. Similarly, if the system were comprised of three load sharing components, three life distributions would be required, and for *n* components in a load sharing configuration, *n* life distributions would need to be obtained. Clearly as the number of components in a load sharing configuration increases so does the number of tests, data and analyses. But is this really the case? The solution to this problem is to use accelerated life data models. Accelerated life tests provide the data from which a life-stress relationship of a product can be obtained.

The value of accelerated life tests extends beyond the need to reduce testing time. One piece of information they can provide, which is commonly overlooked, is the fact that in an accelerated test, life is associated with stress. In other words, an accelerated test can also be described as a life-stress test, which can provide very valuable information regarding the performance of the product under different and changing operating conditions, such as in load sharing redundancy.

In this paper, we examine two different problems related to load sharing components. First, we investigate the problem of analyzing data obtained from a system with load sharing components. When collecting data from such systems, the applied stress on each component in the system is time-dependent, and it is based on the states of the rest of the components. Analysis of this type of data requires the use of a method that takes into account the stress profile of each component, in order to estimate their reliabilities. Once the stress profile has been determined for each component, and by utilizing the data (times-to-failure and suspensions), the reliability of each component can be determined using the cumulative damage model. The second part of the problem deals with solving for the reliability of the system.

## 2. NOTATION

| | |
|---|---|
| $\beta$ | Weibull shape parameter |
| $\eta$ | Weibull scale parameter |
| $T_F$ | exact time-to-failure |
| $T_S$ | right censored time-to-failure |
| x(t) | stress as a function of time |

$B$ parameter of the Arrhenius relationship associated with the activation energy
$C$ parameter of the Arrhenius relationship
$a$ parameter of the IPL relationship
$n$ parameter of the IPL relationship
$t_e$ equivalent operating time of a unit if it had been operating at a different stress level
$t_p$ life characteristic
$S$ stress
$A$ parameter of the Eyring relationship
$f()$ probability density function
$R()$ reliability function
$R'()$ reliability function at an increased stress
IPL Inverse Power Law
MLE Maximum Likelihood Estimation method
QALT Quantitative Accelerated Life Tests

## 3. BACKGROUND THEORY

### 3.1 *Load Sharing Reliability*

For simplicity purposes, we will consider two load sharing components. If we assume that the two components are statistically independent, then the reliability of a simple parallel case is given by:

$$R(t) = 1 - (1 - R_1(t))(1 - R_2(t)). \quad (1)$$

However since these two components are not independent, eq. (1) is no longer valid. Kececioglu (Ref. 2) gives the reliability equation for two load sharing components as follows:

$$\begin{aligned} R(t) &= R_1(t) \cdot R_2(t) \\ &+ \int_0^t f_1(x) \cdot R_2(x) \cdot \frac{R_2'(t_{1e} + (t-x))}{R_2'(t_{1e})} dx \quad (2) \\ &+ \int_0^t f_2(x) \cdot R_1(x) \cdot \frac{R_1'(t_{2e} + (t-x))}{R_1'(t_{2e})} dx \end{aligned}$$

$R_1'()$ and $R_2'()$ are the reliability functions of each component at the increased stress level. Eq. (2) can easily be expanded for $n$ load sharing components. Kececioglu (Ref. 2) points out that the distribution of each component at the increased stress level can be determined through testing at that stress. This is the same as performing a Quantitative Accelerated Life Test. A distribution and a life-stress relationship can then be fitted to the data, thus obtaining a failure distribution as a function of stress. Therefore, eq. (2) can be rewritten in terms of the reliability of each component as a function of stress:

$$\begin{aligned} R(t,S) &= R_1(t,S_1) \cdot R_2(t,S_2) \\ &+ \int_0^t f_1(x,S_1) \cdot R_2(x,S_2) \cdot \frac{R_2(t_{1e} + (t-x), S)}{R_2(t_{1e}, S)} dx \quad (3) \\ &+ \int_0^t f_2(x,S_2) \cdot R_1(x,S_1) \cdot \frac{R_1(t_{2e} + (t-x), S)}{R_1(t_{2e}, S)} dx \end{aligned}$$

where $S_1$ and $S_2$ are some portion of the total stress, $S$, and $R(t,S)$ is the reliability as a function of stress. For example, assuming a Weibull distribution and the IPL life-stress relationship the reliability of a component is given by:

$$R_i(t,S) = e^{-\left(t \cdot a^{-n} \cdot S^n\right)^\beta}. \quad (4)$$

Eq. (4) is substituted into eq.(3), and the reliability of the load sharing system can then be calculated. The benefit of this formulation is that it allows the calculation of the reliability of a load sharing system for different stresses, as well as for different stress allocations for each component. For example, the total stress might be allocated as 70% for component 1 and 30% for component 2 instead of 50% for each component. Also, this formulation can easily be expanded to multiple load sharing components, without requiring additional data or analyses.

### 3.2 *Quantitative Accelerated Life Data Models*

Accelerated life models consist of an underlying failure distribution and a life-stress relationship. A percentile of the failure distribution is chosen to be represented by the stress-life relationship. The objective then becomes to obtain the parameters of the failure distribution and the life-stress relationship. Table 1 presents some common life-stress relationships.

Table 1: Common Life-Stress Relationships

| Relationship | Model |
|---|---|
| Arrhenius | $t_p = Ae^{\frac{B}{S}}$ |
| Eyring | $t_p = \frac{1}{S} e^{-\left(A - \frac{B}{S}\right)}$ |
| Inverse Power | $t_p = \left(\frac{a}{S}\right)^n$ |

The life characteristic, $t_p$, can represent any percentile of the distribution. The percentile is chosen according to the assumed underlying distribution. Some typical life characteristics are presented in Table 2 (Ref. 3).

Table 2: Typical life characteristics (β and σ are assumed to be constant)

| Distribution | Parameters | Life Characteristic |
|---|---|---|
| Weibull | β, η | Scale Parameter (η) |
| Exponential | λ | Mean Life (1/λ) |
| Lognormal | μ, σ | Median ( $\breve{T}$ ) |

The objective then becomes to obtain the parameters of the failure distribution and the life-stress relationship. For example, the Weibull-IPL model is:

$$f(t,S) = \beta \cdot a^{-n} \cdot S^n \left(t \cdot a^{-n} \cdot S^n\right)^{\beta-1} e^{-\left(t \cdot a^{-n} \cdot S^n\right)^\beta}.$$

The parameters to be estimated are $\beta$, $a$, and $n$. In this paper the MLE method will be utilized for estimating the model parameters.

### 3.3 *Cumulative Damage*

Nelson (Ref.1) describes the cumulative damage model. A small overview of the model for any time-dependent stress is given in this section. The model will be formulated using the Weibull distribution. For the Weibull distribution it is assumed that the scale parameter, $\eta$, is a function of stress.

In the case of a time-dependent stress, the scale parameter is also a function of time. In this case, the cumulative damage-Weibull reliability function is given by (Ref.1):

$$R(t,x(t)) = e^{-\left[\int_0^t \frac{1}{\eta(u,x(u))}du\right]^{\beta}}. \quad (5)$$

Any life-stress relationship can be used to express $\eta$ as a function of stress. For example, for the IPL-Weibull cumulative damage model, eq. (5) can be written as

$$R(t,x(t)) = e^{-\left[\int_0^t \left(\frac{x(u)}{a}\right)^n du\right]^{\beta}}. \quad (6)$$

Eq. (6) can be used to estimate the reliability of a product at a given time under a time-dependent stress, and given the parameters $\beta$, $a$ and $n$.

### 3.3.1 *Parameter Estimation*

The parameters of the cumulative damage model can be estimated using the maximum likelihood estimation method, from data obtained from either a constant stress test or a time-dependent stress test. A general likelihood function can be formulated, which will include both cases of time-dependent and constant stress data. The likelihood function for the cumulative damage IPL-Weibull model is given by (Ref. 3):

$$\Lambda = \sum_{i=1}^{F} \ln\left\{\beta \cdot \left[\frac{x(T_{F,i})}{a}\right]^n \cdot \left[\int_0^{T_{F,i}} \left(\frac{x(u)}{a}\right)^n du\right]^{\beta-1}\right\} - \sum_{i=1}^{F}\left\{\left[\int_0^{T_{F,i}} \left(\frac{x(u)}{a}\right)^n du\right]^{\beta}\right\} - \sum_{j=1}^{S}\left\{\left[\int_0^{T_{S,j}} \left(\frac{x(u)}{a}\right)^n du\right]^{\beta}\right\}. \quad (7)$$

Note in eq. (7) that any stress profile, *x(t)*, can be used. Nelson (Ref.1) identifies the following types of time-dependent stress loading as some of the most commonly used in practical applications.

1. Step stress.
2. Ramp stress.
3. Cyclical stress.
4. Randomly varying stress over time (stochastic loading).
5. Non-repeating pattern.

The algorithm used in this paper is independent of the form of the stress profile. In other words, engineers can use different stress profiles and perform the analysis for a wide range of stress profiles, including the above-mentioned stress loading, as well as constant stress loading scenarios.

## 4. APPLICATIONS

### 4.1 *Analyzing Load Sharing System Data*

This example will examine the analysis of data obtained from a system with load sharing components. The data presented are from a non-repairable system with two load sharing components, A and B. The two components are not the same part number, and therefore need to be analyzed separately.

The particular data represent real-life data from a fielded system, but have been modified for the purposes of this paper. This by no means affects the methodology and analyses to be presented. To further mask the data, the actual stresses have been replaced by percentages.

Table 3: Test data

| System | Time to First Event | Event Description | Time to System Failure |
|---|---|---|---|
| System 1 | 65 | B Failed (A Operating alone) | 102 |
| System 2 | 84 | A Failed (B Operating alone) | 148 |
| System 3 | 88 | A Failed (B Operating alone) | 202 |
| System 4 | 121 | B Failed (A Operating alone) | 156 |
| System 5 | 123 | B Failed (A Operating alone) | 148 |
| System 6 | 139 | A Failed (B Operating alone) | 150 |
| System 7 | 156 | B Failed (A Operating alone) | 245 |
| System 8 | 172 | B Failed (A Operating alone) | 235 |
| System 9 | 192 | B Failed (A Operating alone) | 220 |
| System 10 | 207 | A Failed (B Operating alone) | 214 |
| System 11 | 212 | B Failed (A Operating alone) | 250 |
| System 12 | 212 | A Failed (B Operating alone) | 220 |
| System 13 | 213 | A Failed (B Operating alone) | 265 |
| System 14 | 220 | A Failed (B Operating alone) | 275 |
| System 15 | 243 | A Failed (B Operating alone) | 300 |
| System 16 | 248 | B Failed (A Operating alone) | 300 |
| System 17 | 257 | A Failed (B Operating alone) | 330 |
| System 18 | 263 | A Failed (B Operating alone) | 350 |

The data in **Table 3** will be analyzed using the cumulative damage model. To do so, the stress profile for each component needs to be defined. For example, component A of system 1 operated from 0 to 65 hours at a 50% stress level and from 65 to 102 hours at a 100% stress level. **Figure 1** illustrates the stress profile for component A of system 1.

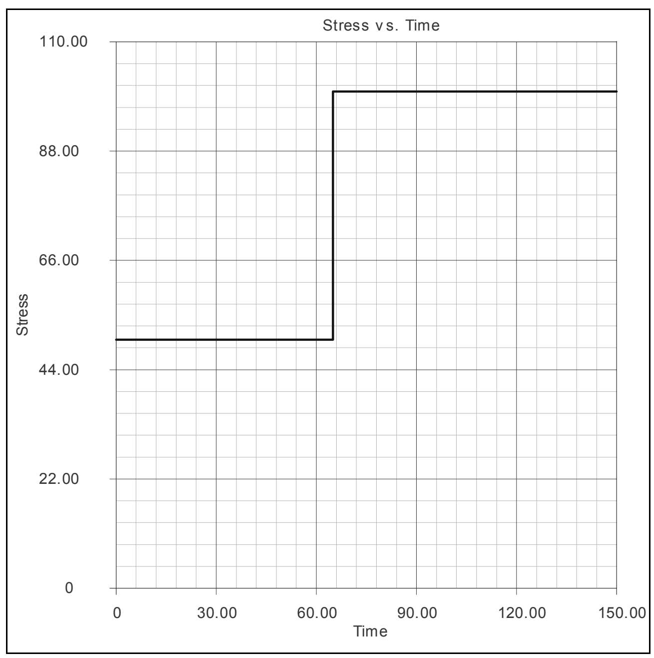


Figure 1: Stress Profile of Component A of System 1

The stress profiles of all the components of all the systems can be defined similarly. In this example, a total of 19 profiles were defined. If this were a repairable system, then the stress would return to 50% after the repair and return to service of component B. So the only additional complexity would have been a more extensive stress profile for each component, but the same methodology and analysis would apply.

Once the stress profiles for each component have been defined, the data can be analyzed using the cumulative damage model (Ref. 5).

For component A, the parameters of the Weibull-IPL model are:

$\beta = 3.3248$, $a = 9836.5892$ and $n = 1.0424$.

For component B the parameters of the Weibull-IPL model are:

$\beta = 2.4995$, $a = 7737.5616$ and $n = 1.1129$.

Now that the life distributions of the two components have been obtained, the reliability of the system can be estimated. For example, the load sharing system's reliability at 200 hours can be obtained by substituting these parameters into eqs. (3) and (4). Solving eq. (3), with $S=100$, $S_1=50$ and $S_2=50$ yields (Ref. 6):

$$R(180) = 77.34\% .$$

For comparison purposes, if the two components were assumed to be statistically independent, and each to supply 50% of the load, then the reliability of the simple parallel system would be:

$$R(180) = 1 - (1 - R_1(180,50)) * (1 - R_2(180,50)) = 84.48\% .$$

This result is significantly different than the 77.34% calculated when taking into account the increased load for the surviving component. **Figure 2** compares the probability of failure for this system, estimated using eq. (3), with the simple parallel system case.

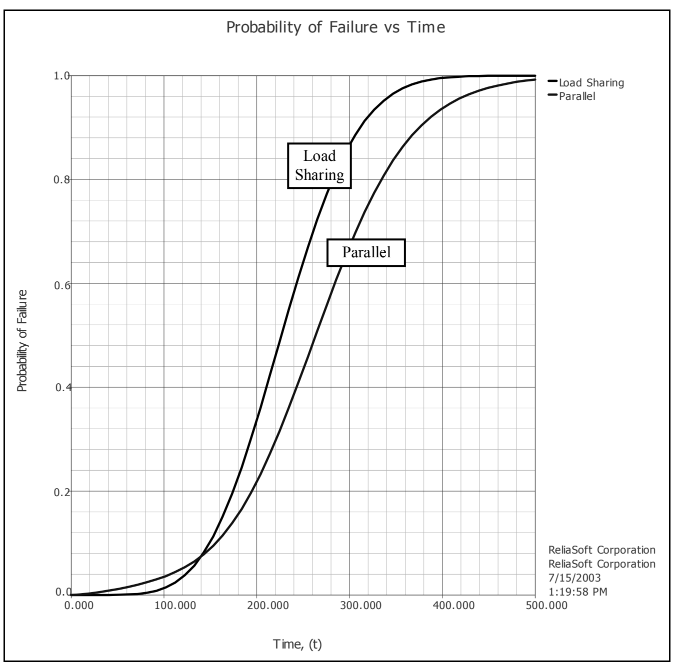


Figure 2: Comparison of Load Sharing Analysis vs. Simple Parallel Analysis

It is obvious from **Figure 2** that the reliability of this system would have been significantly overestimated if the components were assumed to be statistical independent. Using eq. (3) a more realistic estimate was obtained. Simulation methods can also be used to obtain the reliability of load sharing components, especially when dealing with repairable systems. In those cases, a distribution and a life-stress relationship must still be obtained for each load-sharing component (as illustrated in this paper), however, eq. (3) will no longer be applicable.

## ACKNOWLEDGMENTS

We give special thanks to all ReliaSoft personnel for their support and feedback on this paper.

## REFERENCES

1. Nelson, W., Accelerated Testing: Statistical Models, Test Plans, and Data Analyses, 1990; John Wiley & Sons.
2. Kececioglu, D., Reliability Engineering Handbook, Vol. 2, 1991, PTR Prentice Hall.
3. Mettas, A. and Vassiliou, P., "Modeling and Analysis of Time-Dependent Stress Accelerated Life Data," RAMS Proceedings, 2002 Jan.
4. ReliaSoft Corporation, Weibull++ Version 6.
5. ReliaSoft Corporation, ALTA™ Version 6.
6. ReliaSoft Corporation, BlockSim™ Version 6.

## BIOGRAPHIES

Adamantios Mettas
ReliaSoft Corporation
Corporate R&D
115 S. Sherwood Village Dr.
Tucson, AZ 85710 USA
adamantios.mettas@ReliaSoft.com

Mr. Mettas is the Senior Research Scientist at ReliaSoft Corporation. He fills a critical role in the advancement of ReliaSoft's theoretical research efforts and formulations in the subjects of Life Data Analysis, Accelerated Life Testing, and System Reliability and Maintainability. He has played a key role in the development of ReliaSoft's software including Weibull++, ALTA, and BlockSim, and has published numerous papers on various reliability methods. Mr. Mettas holds a BS Degree in Mechanical Engineering and an MS degree in Reliability Engineering from the Univ. of Arizona.

Pantelis Vassiliou
ReliaSoft Corporation
Corporate R&D
115 S. Sherwood Village Dr.
Tucson, AZ 85710 USA
pantelis.vassiliou@ReliaSoft.com

Mr. Vassiliou is President and CEO of ReliaSoft Corporation, an industry leader in reliability engineering R&D, software and services that combine the latest theoretical advances with essential tools for the practitioner in the field. He is a renowned expert and lecturer on reliability engineering and ReliaSoft's founder. Mr. Vassiliou holds a B.S. degree in Aeronautical Engineering from Embry-Riddle and an M.S. degree in Reliability Engineering from the Univ. of Arizona.