An Extended Reliability Growth Model For Managing And Assessing Corrective Actions

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Key Words: Reliability growth, Crow (AMSAA) model, Projections, Extended reliability growth model, Maturity metrics

SUMMARY & CONCLUSIONS

The most widely used traditional reliability growth tracking model and reliability growth projection model are both included as IEC International Standard and US ANSI National Standard models. These traditional models address reliability growth based on failure modes surfaced during the test. With the tracking model all corrective actions are incorporated during test, called test-fix-test. With the projection model all corrective actions are delayed until the end of test. This is called test-find-test. However, the most common approach for development-testing programs include some corrective actions during testing and some delayed fixes incorporated at the end of test. That is, test-fix-find-test. This paper presents an Extended Model that addresses this practical situation and allows for preemptive corrective actions.

1. INTRODUCTION

In today's environment of compressed schedules and limited testing, every opportunity to identify and correct reliability deficiencies in a new design is a prime objective. A metric for tracking system reliability before development testing based on preemptive corrective actions for potential problem modes is discussed in Ref. 1. The tradition reliability growth models, (See, for example, Ref. 2, Ref. 3, Ref. 4, Ref. 5) provide assessments when the failure modes corrected are surface during the testing. In the test-fix-test strategy problem modes are found during testing and corrective actions for these problems are incorporated during the test. For the test-find-test strategy problem modes are found during testing but all corrective actions for these problems are delayed and incorporated after the completion of the test. The focus of this paper is a combination of these two approaches, referred to as test-fix-find-test. This is the practical situation where some corrective actions are incorporated during the test and some corrective actions are delayed until the end of the test. Reliability growth assessments for the test-fix-find-test strategy are not provided with the widely used tradition models in international and U. S. ANSI standards.

This paper presents an extended reliability growth model that provides assessments for the test-fix-find-test strategy and also allows for preemptive corrective actions. The Extended Model preserves the properties of the traditional models and reduces to these models and strategies as special cases. The model also provides extensive metrics useful for managing the reliability program. The preemptive corrective actions strategy, the Extended Model, and the maturity metrics are all currently being used in industry and government. As a practical management methodology, the model parameters and management metrics are simple and straightforward to calculate and interpret. These applications will be illustrated by numerical examples.

In Section 2 we will provide background on the widely used test-fix-test and test-find-test models. In Section 3 we present the Extended Model for test-fix-find-test applications and in Section 4 we give management and maturity metrics. In Section 5 we note a number of current applications of the Extended Model and associated metrics.

Notation

λ	Scale parameter for Crow (AMSAA) model
β	Shape parameter for Crow (AMSAA) model
α	Growth rate for Crow (AMSAA) model
t	Test time
T	Total test time
MTBF	Mean time between failures
$r(\cdot)$	Crow (AMSAA) model failure intensity
X_i	The i-th successive failure time
N	Total number of failures
λ_A	Type A modes failure intensity
λ_B	Type B modes failure intensity
λ_P	Projected failure intensity
M_P	Projected MTBF
$h(\cdot)$	Rate of uncovering new failure modes
λ_{GP}	Growth potential failure intensity
M_{GP}	Growth potential MTBF
λ_{EM}	Extended model failure intensity
M_{EM}	Extended model MTBF
$\Gamma(\cdot)$	Gamma function

2. BACKGROUND

To lay the groundwork for the Extended Model we first give some background on the two widely used basic models.

2.1. CROW (AMSAA) Basic Model for Test- Fix- Test

The Duane postulate, Ref.7, for reliability growth during test-fix- test development testing states that the instantaneous system MTBF at cumulative test time t is $M(t) = \left[\lambda \beta t^{\beta-1}\right]^{-1}$, where 0 < λ and 0< β are parameters.

Crow (Ref.2) modeled the Duane postulate stochastically as a non-homogeneous Poisson process (NHPP) with intensity

$$r(t) = \lambda \beta t^{\beta-1}, \tag{1}$$

thus allowing for statistical procedures based on this process for reliability growth analyses. This model is applicable to test-fix-test data not test-fix-find-test. Estimation procedures, confidence intervals, etc. are given in Reference 4.

The parameter λ is referred to as the scale parameter and β is the shape parameter. For $\beta = 1$, there is no reliability growth. For $\beta < 1$, there is positive reliability growth. That is, the system reliability is improving due to corrective actions. For $\beta > 1$, there is negative reliability growth.

Under the Crow (AMSAA) basic model the achieved or demonstrated failure intensity at time T, the end of the test, is given by r(T). We denote the achieved failure intensity by

$$\lambda_{CA} = r(T). \tag{2}$$

Suppose a development testing program begins at time 0 and is conducted until time T and stopped. Let N be the total number of failures recorded and let $0<X_1 < X_2 < \ldots X_N < T$ denote the N successive failure times on a cumulative time scale. We assume that the Crow (AMSAA) NHPP assumption applies to this set of data. Under the Crow (AMSAA) basic model the maximum likelihood estimates (MLEs) for λ and β (numerator of MLE for β adjusted from N to N-1 to obtain unbiased estimate) are

$$\hat{\lambda} = \frac{N}{T^{\hat{\beta}}} \quad , \quad \hat{\beta} = \frac{N-1}{\sum_{i=1}^{N} \frac{T}{X_i}} \quad . \tag{3}$$

Example 1. Test-Fix-Test

To illustrate the general application of this model consider a system tested for T=400 hours with the 56 failure times given in Table 1. The first failure was recorded at .7 hours into the test, the second failure was recorded 3 hours later at 3.7. The last failure occurred at 395.2 hours into the test, and the testing was stopped 4.8 hours later at 400.

0.7	63.6	125.5	244.8	315.4	*366.3*
3.7	72.2	133.4	249	317.1	*373*
13.2	99.2	151	250.8	320.6	*379.4*
15	99.6	163	260.1	324.5	*389*
17.6	100.3	164.7	263.5	324.9	*394.9*
25.3	102.5	174.5	273.1	342	*395.2*
47.5	112	177.4	274.7	350.2	
54	112.2	191.6	282.8	355.2	
54.5	120.9	192.7	285	364.6	
56.4	121.9	*213*	*304*	*364.9*	

Table 1. Test-Fix-Test Data

Applying equations (3) we get the estimates

$$\hat{\lambda} = 0.2397 \qquad \hat{\beta} = 0.9103 \quad . \tag{4}$$

The achieved or demonstrated failure intensity and MTBF are estimated by

$$\hat{\lambda}_{CA} = \hat{\lambda}\hat{\beta} T^{\hat{\beta}-1} \text{ or } \hat{\lambda}_{CA} = 0.1274 \ , \tag{5}$$

$$\hat{M}_{CA} = \left[\hat{\lambda}_{CA}\right]^{-1} = 7.84. \tag{6}$$

It is important to note that the Crow (AMSAA) test-fix test model does not assume that all failures in the data set (e.g. Table 1) receive a corrective action. Based on the management strategy some failures may receive a corrective action and some may not. This is discussed next, in Section 2.2, and addressed throughout the remainder of this paper.

2.2 CROW (AMSAA) Projection Model for Test- Find -Test

Suppose a system is tested for time T. During the testing problem failure modes are identified, but all corrected actions are delayed and incorporated at the end of the test phase. This is test-find-test. These delayed corrective actions are usually incorporated as a group and the result is generally a distinct jump in the system reliability. The projection model, Ref. 5, estimates this jump in reliability due to the delayed fixes. This is called a "projection."

The projection model places all failure into two groups, A and B. Type A failure modes are all modes such that if seen during test no corrective action will be taken. This accounts for all modes for which management determines that it is not cost-effective to increase the reliability by a design change. Type B failure modes are all modes such that if seen during test a corrective action will be taken. This Type A and Type B determination helps define the reliability growth management strategy. The basic projection model assumes that the Type A failure modes has constant failure intensity λ_A, the i-th Type B failure mode follows the exponential distribution with failure rate λ_i, and the initial failure intensity for Type B failure modes is λ_B.

Example 2. Test-Find-Test

For the data in Table 2 the system is tested for T=400 hours. There is a total of N=42 failures and all corrective actions will be incorporated at the end of the 400 hour test. Each failure is designated as either a Type A failure mode (no corrective action) or Type B failure mode (corrective action). There are $N_A = 10$ Type A failures and $N_B = 32$ Type B failures during the test.

In Table 2 there are M = 16 unique Type B failure modes seen which means there are 16 distinct corrective actions incorporated into the system at the end of test. The total number of failures for the j-th observed distinct Type B mode is denoted by N_j and the total number of Type B failures seen during the test is $N_B = \sum_{j=1}^{M} N_j$. See Table 3.

j	X_j	Mode	j	X_j	Mode
1	15	B1	22	260.1	B1
2	25.3	B2	23	263.5	B8
3	47.5	B3	24	273.1	A
4	54	B4	25	274.7	B6
5	56.4	B5	26	285	B13
6	63.6	A	27	304	B9
7	72.2	B5	28	315.4	B4
8	99.6	B6	29	317.1	A
9	100.3	B7	30	320.6	A
10	102.5	A	31	324.5	B12
11	112	B8	32	324.9	B10
12	120.9	B2	33	342	B5
13	125.5	B9	34	350.2	B3
14	133.4	B10	35	364.6	B10
15	164.7	B9	36	364.9	A
16	177.4	B10	37	366.3	B2
17	192.7	B11	38	373	B8
18	213	A	39	379.4	B14
19	244.8	A	40	389	B15
20	249	B12	41	394.9	A
21	250.8	A	42	395.2	B16

Table 2: Test-Find-Test Data

B Mode j	Number N_j	First Occurrence	EF d_j
1	2	15.0	.67
2	3	25.3	.72
3	2	47.5	.77
4	2	54.0	.77
5	3	56.4	.87
6	2	99.6	.92
7	1	100.3	.50
8	3	112.0	.85
9	3	125.5	.89
10	4	133.4	.74
11	1	192.7	.70
12	2	249.0	.63
13	1	285.0	.64
14	1	379.4	.72
15	1	389.0	.69
16	1	395.2	.46

Table 3: Test-Find-Test Type B Failure Mode Data and Effectiveness Factors

An effectiveness factor (EF) d_j is the fraction decrease in λ_j after a corrective action has been made for the j-th Type B mode. The failure rate for the i-th Type B failure mode after a corrective action is (1-d_j)λ_j. In practice, for application of the projection model, the EFs are assigned based on engineering assessments, test results, etc. Studies indicate that an average EF d of about .70 is typical for a reliability growth program. Individual EFs may vary. The assigned EFs for distinct Type B modes are given in Table 3.

For test-find-test the system failure intensity is constant, say, λ_S, during the testing and then jumps to a lower value due to the incorporation of corrective actions. The intensity at the end of the test T, before delayed corrective actions are introduced into the system, is the achieved intensity. The reciprocal of the intensity is the achieved MTBF M_S.

We estimate the achieved failure intensity λ_S by

$$\hat{\lambda}_S = \hat{\lambda}_A + \hat{\lambda}_B\,, \quad \hat{\lambda}_A = N_A/T\,, \quad \hat{\lambda}_B = N_B/T\,. \tag{7}$$

Based on the data in Table 2,

$$\hat{\lambda}_S = 0.105 \quad \hat{\lambda}_A = 0.025\,, \quad \hat{\lambda}_B = 0.08\,. \tag{8}$$

The estimated achieved MTBF M_S at time T = 400 before the jump is $\hat{M}_S = 9.5$. We estimate the jump next.

The estimated projected failure intensity, Ref. 5, is

$$\hat{\lambda}_P = \hat{\lambda}_A + \sum_{j=1}^{M}(1-d_j)\frac{N_j}{T} + \bar{d}\,\hat{h}(T) \tag{9}$$

where $$\bar{d} = \frac{\sum_{j=1}^{M} d_j}{M}\,,$$

is the average EF, and

$$\hat{h}(T) = \hat{\lambda}\hat{\beta}T^{\hat{\beta}-1}\,. \tag{10}$$

The projection model $\hat{\lambda}$ and $\hat{\beta}$ for (10) use only the M first occurrence failure times of the seen and unique Type B failure modes. These first occurrences are given in Table 3. Applying equations (3) to the first occurrence data in Table 3 we have

$$\hat{\lambda} = 0.1820\,, \quad \hat{\beta} = 0.7472\,. \tag{11}$$

Based on the data in Tables 2 and 3, we have M = 16,

$$\bar{d} = .72\,, \quad \hat{h}(400) = 0.0299. \tag{12}$$

Also,

$$\bar{d}\,\hat{h}(T) = 0.0215\,, \quad \hat{\lambda}_A = 0.025$$

$$\sum_{j=1}^{M}(1-d_j)\frac{N_J}{T} = 0.0196\,. \tag{13}$$

From (9) the projected failure intensity and MTBF are

$$\hat{\lambda}_p = 0.0661\,, \quad \hat{M}_p = 15.1. \tag{14}$$

The projection model estimates that the MTBF jumps to 15.1 hours from 9.5 hours due to the 16 distinct corrective actions.

3. EXTENDED RELIABILITY GROWTH MODEL FOR TEST-FIX–FIND-TEST

In order to provide the assessment and management metric structure for corrective actions during and after a test, we define two types of B modes. Type BC failure modes are corrected during test. Type BD failure modes are delayed to the end of the test. Type A failure modes, as before, are those failure modes that will not receive a corrective action. These define the management strategy and can be changed. The Crow (AMSAA) basic model (Ex. 1) does not utilize the failure mode designation. This BC and BD failure mode designated is an important aspect of the Extended Model.

The test-fix-find-test concept is illustrated in Table 4. This is the same data as in Table 1, but Table 4 denotes those failure modes that received a corrective action during the test (BC modes) and also those failure modes that will receive a corrective action at the end of the test (BD modes). The test-fix-find-test strategy in Table 4 is to fix more failure modes than with the test-fix-test management strategy. During test the Type A and Type BD failure modes do not contribute to reliability growth. The corrective actions for the BC failure modes affect the increase in the system reliability during the test and this is the same for both the Table 1 and Table 4 management strategy. After the incorporation of corrective actions for the Type BD failure modes, the reliability increases. Estimating this increased reliability with test-fix-find-test data is the objective of this paper.

For the Extended Model we assume that the achieved MTBF, before delayed fixes, based on Table 4 data should be exactly the same as the achieved MTBF for the Table 1 data. If K is the total number of distinct BD modes then, in Ref.5, the intensity to be estimated is

$$\lambda_P = \lambda_S - \lambda_B + \sum_{i=1}^{K}(1-d_i)\lambda_i + dh(T) \,. \tag{15}$$

To allow for BC failure modes in the extended model we replace λ_S by λ_{CA} in (15). Also, let λ_{BD} be the constant failure intensity for the Type BD failure modes, and let $h(t|BD)$ be the first occurrence function for the Type BD failure modes (see (10).

The Extended Model projected failure intensity is

$$\lambda_{EM} = \lambda_{CA} - \lambda_{BD} + \sum_{i=1}^{K}(1-d_i)\lambda_i + dh(T|BD) \,. \tag{16}$$

The Extended Model projected MTBF is $M_{EM} = 1/\lambda_{EM}$. This is the MTBF after the incorporation of the delayed BD failure modes that we wish to estimate.

Under the Extended Model the achieved failure intensity, before the incorporation of the delayed BD failure modes, is the first term λ_{CA}.The achieved MTBF at time T before the BD failure modes is $M_{CA} = [\lambda_{CA}]^{-1}$. That is, the achieved MTBF before delayed fixes for the data in Table 4 is exactly the same as the achieved MTBF for the data in Table 1.

3.1. Estimation for the Extended Reliability Growth Model

The estimate of the projected failure intensity for the Extended Model is

$$\hat{\lambda}_{EM} = \hat{\lambda}_{CA} - \hat{\lambda}_{BD} + \sum_{j=1}^{M}(1-d_j)\frac{N_j}{T} + \bar{d}\,\hat{h}(T|BD) \tag{17}$$

where the first term is the Crow (AMSAA) model estimate (see (5)) applied to all A, BC, and BD data, as in Example 1, and the remaining terms are calculated as in Example 2, using only the BD data in Table 3 and Table 4.

If it is assumed that no corrective actions are incorporated into the system during the test (no BC failure modes), then this is equivalent to assuming that $\beta = 1$ for $\lambda_{CA,}$, (see (5)), and λ_{CA} is estimated as in Example 2. In general, the assumption of a constant failure intensity ($\beta = 1$) can be assessed by a statistical test from the data.

Example 3. Test-Fix-Find-Test

In Table 4 there are 56 total failures, T=400, and the failure times are the same as in Example 1. Table 4 will be used for several examples. For the current example assume that all the failure times X_J are known. There are BC failure modes but assume in this example that only the BD failure modes are designated. Assume the remaining Type A and Type BC failure modes are not designated as such.

The first term, $\hat{\lambda}_{CA}$, in $\hat{\lambda}_{EM}$ (Eq. (17) uses all failure time data in Table 4, as in Example 1. This gives (see (5))

$$\hat{\lambda}_{CA} = 0.1274 \,. \tag{18}$$

For the remaining terms in Eq. (17) the BD data in Table 4, and the EFs given in Table 3, are used. This Type BD data is the same Type B data in Example 2 so the calculations in Eq. (17) are the same. That is,

M=16, T= 400, $\lambda_{BD} = 0.08, \bar{d} = 0.72$. Also,

$$\sum_{j=1}^{M}(1-d_J)\frac{N_J}{T} = 0.0196 \,, \tag{19}$$

and

$\hat{h}(T|BD)$ has parameters $\hat{\lambda} = 0.1820$, $\hat{\beta} = 0.7472$.

This gives $\bar{d}\,h(T|BD) = 0.0215$. Therefore,

$$\hat{\lambda}_{EM} = 0.1274 - 0.08 + 0.0196 + 0.0215 \tag{20}$$

or

$$\hat{\lambda}_{EM} = 0.0885. \tag{21}$$

The Extended Model projected MTBF is $\hat{M}_{EM} = 11.29$. The achieved MTBF before the 16 delayed fixes is estimated by $\hat{M}_{CA} = 7.84$. We therefore have based on the Extended Model estimates that the MTBF grew to 7.84 as a result of corrective actions for BC failure modes during the test, and then jumped to 11.29 as a result of the delayed corrected actions after the test for the BD failure modes.

J	X_J	Mode	J	X_J	Mode
1	0.7	BC1	29	192.7	BD11
2	3.7	BC1	30	213	A
3	13.2	BC1	31	244.8	A
4	15	BD1	32	249	BD12
5	17.6	BC2	33	250.8	A
6	25.3	BD2	34	260.1	BD1
7	47.5	BD3	35	263.5	BD8
8	54	BD4	36	273.1	A
9	54.5	BC3	37	274.7	BD6
10	56.4	BD5	38	282.8	BC11
11	63.6	A	39	285	BD13
12	72.2	BD5	40	304	BD9
13	99.2	BC4	41	315.4	BD4
14	99.6	BD6	42	317.1	A
15	100.3	BD7	43	320.6	A
16	102.5	A	44	324.5	BD12
17	112	BD8	45	324.9	BD10
18	112.2	BC5	46	342	BD5
19	120.9	BD2	47	350.2	BD3
20	121.9	BC6	48	355.2	BC12
21	125.5	BD9	49	364.6	BD10
22	133.4	BD10	50	364.9	A
23	151	BC7	51	366.3	BD2
24	163	BC8	52	373	BD8
25	164.7	BD9	53	379.4	BD14
26	174.5	BC9	54	389	BD15
27	177.4	BD10	55	394.9	A
28	191.6	BC10	56	395.2	BD16

Table 4. Test-Fix-Find-Test Failure Times and Failure Mode Designations

3.2. Extended Reliability Growth Model with Preemptive Corrective Actions

Suppose that in addition to delayed fixes, there are also preemptive fixes at time T for failure modes that have not experienced a failure. We assume that these failure modes are in fact Type B modes in the sense that they would have received a corrective action if they had occurred during the test. Let Q be the number of preemptive failure modes receiving a fix at time T, and denote these by $BP_1, BP_2, ..., BP_Q$. For each of these failure modes we assume that we have an estimate (by analysis, analogy, or test) of the failure rate, λ_q before the corrective action, and estimate of the corresponding effectiveness factor, d_q, q = 1,…,Q . That is, the failure rate is estimated by λ_q before the corrective action and by $(1-d_q)\lambda_q$ after the corrective action. The Extended Model failure intensity estimate, λ^*_{EM}, with the Q preemptive corrective actions is

$$\lambda^*_{EM} = \hat{\lambda}_{EM} - \sum_{q=1}^{Q} d_q \lambda_{BP_q} \quad . \qquad (22)$$

Example 4. Test-Fix-Find-Test with Preemptive Corrective Actions

Consider again the previous example but suppose that at the end of the 400 hour test Q=3 preemptive corrective actions were incorporated into the system in addition to the 16 delayed fixes. Let

$$\lambda_{BP_1} = 0.0010, \lambda_{BP_2} = 0.0005, \lambda_{BP_3} = 0.0007$$

$d_{BP_1} = 0.60, d_{BP_2} = 0.85, d_{BP_3} = 0.65$. Then,

$$\hat{\lambda}^*_{EM} = 0.0885 - 0.00159 = 0.0869$$

The corresponding MTBF is $\hat{M}^*_{EM} = 11.50$

.

By incorporating the additional 3 preemptive corrective actions the MTBF jumped from 11.29 to 11.50.

4. EXTENDED MODEL MANGEMENT AND MATURITY METRICS

The Crow (AMSAA) basic model application in Example 1 did not use failure modes designation but the projection model in Example 2 did. Specific knowledge on the BC and BD modes is very informative and the structure of the Extended Model utilizes this information to provide a variety of useful metrics to management and engineering. In this section we give some key Extended Model metrics for some important practical situations. These are illustrated using the data in the examples.

4.1. Test-Fix-Test with BC Failure Modes Known

This is the situation where there are no delayed fixes (no BD failure modes) and all failure modes are either Type BC or Type A. In applying the Extended Model the Type BC failure modes are specifically designated, and the remainder are Type A failures. In Table 4 there are 56 total failures, 14 Type BC failures, 12 unique Type BC failure modes seen, and 42 Type A failures seen. Fitting the Crow (AMSAA) basic model to only the failure time data (Example 1) we have Metrics 1-4 below. The Extended Model provides additional management metrics, Metrics 5-24, for this case.

METRIC 1. $\beta = 0.9103$. (see (4)).

METRIC 2. Growth rate $\alpha = 1 - \beta = 0.0897$.

METRIC 3. Achieved failure intensity $\lambda_{CA} = 0.1274$, (see (5)).

METRIC 4.Achieved MTBF $\hat{M}_{CA} = 7.84$, (see (6)).

We now introduce, in this paper, the calculation for the initial system MTBF of the Extended model. This is

METRIC 5. Initial System MTBF.

$$M_I = \frac{\Gamma\left(1+\frac{1}{\beta}\right)}{\lambda^{\frac{1}{\beta}}} \qquad (23)$$

Using the estimates in (4) $\hat{M}_I = 5.02$.
At the beginning of the test it is estimated that the initial system MTBF was 5.02. Due to corrective actions it is estimated by Metric 4 that the reliability grew to 7.84 at the end of the 400 hour test.

METRIC 6.Intial system failure intensity $\lambda_I = 1/M_I$.

$\hat{\lambda}_I = 0.1991$.

METRIC 7. Type A modes failure intensity.
All failures except BC modes are considered A modes in this application. $N_A = 42$.

$\hat{\lambda}_A = (42/400) = 0.105$.

METRIC 8.Initial A mode MTBF $M_A = 1/\lambda_A$.

$\hat{M}_A = 9.52$

METRIC 9. Type BC initial failure intensity.
This is (Metric 6 – Metric 7) or

$\hat{\lambda}_{I(BC)} = \hat{\lambda}_I - \hat{\lambda}_A = 0.0941$

METRIC 10. Type BC initial MTBF $M_{I(BC)} = 1/\lambda_{I(BC)}$.

$\hat{M}_{I(BC)} = 10.62$.

That is, the problem Type BC modes had a MTBF of 10.62 at the beginning of the test.

METRIC 11. Type BC end of test failure intensity.
This is (**Metric 3** – **Metric 7)** = 0.0225.

METRIC 12. Type BC end of test MTBF.
This is 1/Metric 11 = 44.48.
That is, the corrective actions increased the Type BC failure modes MTBF from 10.62 (Metric 10) to 44.48.

METRIC 13. Failure intensity $h(T|BC)$ for new Type BC failure modes at end of test T. Apply Equation (10) to the M=12 BC first occurrence times in Table 4. This gives

$\hat{\lambda} = 0.3891, \hat{\beta} = 0.5723$ and $\hat{h}(400|BC) = 0.0172$.

This is the rate at which new distinct problem BC failure modes are occurring at the end of the 400 hour test.

METRIC 14. Instantaneous MTBF to next new Type BC failure mode. $M(T|BC) = 1/h(T|BC)$.

$\hat{M}(400|BC) = 58.2$.

New, distinct Type BC failure modes occurred at the beginning of the test with an MTBF of 10.62, and at T = 400, new, distinct Type BC failure modes were occurring every 58.2 hours.

METRIC 15. Average effectiveness factor for Type BC Failure Modes. How effective have the 12 corrective actions been? We introduce, in this paper, the calculation

$$\hat{d}_{BC} = \frac{\hat{\lambda}_I - \hat{\lambda}_{CA}}{\hat{\lambda}_{I(BC)} - \hat{h}(T|BC)}. \qquad (24)$$

For this example, $\hat{d} = 0.81$.
That is, the 12 corrective actions removed an average of 81% of the failure rate from the 12 unique BC failure modes. An average of 19% remained in the 12 BC modes.

METRIC 16. Fraction of system failure intensity not being addressed by corrective actions (Type A modes), and fraction being addressed (Type BC modes)

Fraction Type A = $\dfrac{\hat{\lambda}_A}{\hat{\lambda}_I} = 0.53$.

Fraction Type BC = $\dfrac{\hat{\lambda}_{I(BC)}}{\hat{\lambda}_I} = 0.47$. That is, 53 % (47 %) of the system failure intensity is not (is) being addressed by a corrective action with this A and BC management strategy.

METRIC 17. Growth potential failure intensity with BC modes only. If testing continues with the management strategy in Metric 16 and the effectiveness factor in Metric 15, what is the lowest value attainable for the system failure intensity?
This is,

$\lambda_{GP} = \lambda_A + (1-d)\lambda_{I(BC)}$ and

$\hat{\lambda}_{GP} = 0.1229$. This management strategy can reduce the system failure intensity from 0.1991 (Metric 6) to 0.1229. Current achieved intensity is 0.1274 (Metric 3).

METRIC 18. Growth potential MTBF with BC modes only. Maximum attainable MTBF with management strategy.

$M_{GP} = 1/\lambda_{GP}$. $\hat{M}_{GP} = 8.14$. This management strategy can increase the initial MTBF from 5.02 (Metric 5) to a maximum of 8.14. (Current achieved MTBF is 7.84.) It is important to note that the management strategy can be changed to increase M_{GP}. For example, move existing A modes to BC or BD modes. See Metric 33.

METRIC 19. Intensity for Type BC problem failure modes that have not been seen in the testing. This is an important management maturity metric. The function $h(T|BC)$, given by Metric 13, is also the amount of failure intensity in the system at the end of the test due to the Type BC failure modes that have not been seen in the testing. For this example $\hat{h}(400|BC) = 0.0172$. (not seen)

METRIC 20. Intensity for Type BC failure modes that have been seen in the testing. Metric 9-Metric 19

$\hat{\lambda}_{I(BC)} - \hat{h}(400|BC) = 0.0769$ (seen)

METRIC 21. Fraction of Type BC failure Intensity due to failure modes that have not been seen in the testing.

$\frac{\hat{h}(400|BC)}{\hat{\lambda}_{I(BC)}} = 0.18$ Fraction BC Failure intensity not seen in the testing. (Fraction BC not mature).

METRIC 22. Fraction of Type BC failure intensity due to failure modes that have been seen in the testing. This is,

1-Metric 21 = 0.82. Fraction BC Failure intensity seen in the testing. (Fraction BC mature).

METRIC 23. Fraction of growth potential MTBF achieved at the beginning of testing.

Maturity Factor of Initial design $\frac{\hat{M}_I}{\hat{M}_{GP}} = 0.62$.

METRIC 24. Fraction of growth potential MTBF achieved at current testing.

Maturity Factor of Current design = $\frac{\hat{M}_{CA}}{\hat{M}_{GP}} = 0.96$.

The 12 BC corrective actions have increased the maturity of the design from an initial factor of .62 to .96 relative to the growth potential.

4.2. Test-Find-Test with BD Failure Modes Known

This is the situation discussed in Example 2. There are no Type BC failure modes; only Type A and Type BD delayed fixes. The BD failure modes are designated. Using Table 2 and Table 3 Type BD mode information, $N_A = 10, N_{BD} = 32$.

METRIC 25. Initial Type A and Type BD failure intensities

$\hat{\lambda}_A = \frac{N_A}{T} = \frac{10}{400} = 0.025 \quad \hat{\lambda}_{BD} = \frac{N_{BD}}{T} = \frac{32}{400} = 0.08$

METRIC 26. Intensity for Type BD failure modes that have not been seen in the testing. This is equation (12).

$\hat{h}(400|BD) = 0.0299$.

METRIC 27. Intensity for Type BD failure modes that have been seen in the testing. This is

$\hat{\lambda}_{BD} - 0.0299 = 0.0501$.

METRIC 28. Fraction of Type BD failure intensity due failure modes that have not been unseen (been seen) in test.

Fraction Unseen = (Metric26/ $\hat{\lambda}_{BD}$)=0.37. Seen = 0.63.

METRIC 29. Growth potential failure intensity and MTBF with BD modes only.

$\hat{\lambda}_{GP} = \hat{\lambda}_A + \sum_{j=1}^{M}(1-d_j)\frac{N_j}{T} = 0.0446$ (See Ref.(5))

METRIC 30. Growth potential MTBF with BD modes only. Growth potential MTBF =22.4

4.3 Test-Fix-Find-Test with BD Failure Modes Known

This is the data situation discussed in Example 3. There are corrective actions during the test and at the end of the test. However, assume that specific information on the Type BC failure modes is not given in the data set. In this case only the BD failure modes are designated, so metrics requiring specific knowledge of the BC modes cannot be calculated. However, Metrics 1-6 apply and Metrics 25-28 apply.

4.5 Test Fix Find Test with BC and BD Failure Modes Known

This is the Extended Model situation with all the information in Table 3 and Table 4 known. There are corrective actions during the test and at the end of the test. Specific information on both the Type BC failure modes and Type BD failure modes is given in the data set. In this case Metrics 1-6**,** Metrics 9-15, Metrics 19-22, Metrics 26-28 all apply, plus

METRIC 31. Type A modes failure intensity and MTBF.

In Table 4 $N_A = 10$. $\hat{\lambda}_A = (10/400) = 0.025$

Type A modes MTBF= 40. Fraction Type A = $\frac{\hat{\lambda}_A}{\hat{\lambda}_I} = 0.125$.

METRIC 32. Growth potential failure intensity with both BC and BD failure modes.

$\hat{\lambda}_{GP} = \hat{\lambda}_A + (1-\hat{d}_{BC})\hat{\lambda}_{BC} + \sum_{j=1}^{M}(1-d_j)\frac{N_j}{T} = 0.0624.$

$\bar{d}_{BD} = 0.72$ (See (12).)

METRIC 33. Growth potential MTBF with both BC and BD failure modes.

$\hat{M}_{GP} = 16.0$

This management strategy can increase the initial MTBF from 5.02 (Metric 5) to a maximum of 16.0. The Extended Model projected MTBF is $\hat{M}_{EM} = 11.29$. Compare Metric 33 to Metric 18. The Metric 33 management strategy includes more corrective actions, in this case as Type BD modes, than the Metric 18 management strategy. By changing the management strategy the growth potential MTBF increased from 8.14 to 16.0.

5. EXAMPLES OF EXTENDED RELIABILITY GROWTH MODEL APPLICATIONS

The projection model, Ref. 5, discussed in Ref. 8 for fielded systems application assumes no trend over cycles of operation and overhaul. The use of the Extended Model allows for trends if they occur, and will therefore provided a broader range of applications. In the development of complex, state of the art medical diagnostic equipment, functionality is often added sequentially to the system prototypes during development testing. Consequently the reliability growth for each functionality and associated subsystem is generally managed separately and also at the system level. For each subsystem, the testing is typically test- fix –find- test and, therefore, the Extended Model and metrics apply. These management and maturity metrics are currently being employed for complex medical equipment in order to attain the initial product launch reliability. The preemptive corrective actions methodology discussed in Ref. 1 is being used for a national missile defense program where testing is very expensive and limited by test units. For preemptive corrective actions introduced during missile subsystems development testing, the Extended Model applies. The Extended Model is also being used in the development of complex farm equipment, such as tractors and combines. For these systems there are typically preemptive corrective actions, compressed schedules, and test-fix-find-test development testing. ReliaSoft Corporation has developed software, RGA 6, to implement the application of the Extended Model.

ACKOWLEDGEMENTS

I would like to acknowledge Kevin O'Shaughnessy at the Reliability Analysis Center for his excellent support, and discussions on the application of the Extended Model at Naval Depots. I would also like to thank Adamantios Mettas and David Groebel of ReliaSoft Corporation for the excellent application of the new ReliaSoft reliability growth software in performing the Extended Model analyses in this paper. I wish to also thank Dwight DeDoncker, Quality Services Department at Deere & Company Headquarters, Moline Illinois, for the useful discussions, insight, and extensive applications of the methodology discussed in this paper. I would also like to thank Dough Hearne, Lockheed Martin, Huntsville, AL, for his technical leadership and significant application of the preemptive reliability growth strategy and methods in Ref. 1 to the national missile defense program. I also thank Keith McLain of Advanced Bionics, Sylmar, CA for his pioneering initiative and application of the reliability growth management and maturity metrics to complex, evolving functionality medical equipment development. I also wish to thank the reviewers for their very helpful comments and suggestions.

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