Modeling & Analysis for Multiple Stress-Type Accelerated Life Data

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Key Words: Accelerated Testing, multiple stress-type data, proportional hazards, regression, covariates, multivariable relationships.

SUMMARY & CONCLUSIONS

This paper describes a model for multiple stress-type accelerated life data. In addition, the use of an algorithm, which was specifically developed for this model is illustrated. The model is based on the widely known Log-Linear model (Ref. 4) and is formulated for the Weibull and Lognormal distributions for a variety of censoring schemes using likelihood theory. An algorithm has been developed for the solution of this model, and implemented in a recently released software package, ALTA Pro^{TM}, specific to accelerated life data analysis. The algorithm has been specifically designed to be very flexible and has the capability of simultaneously solving for up to eight different stress-types. The advantage of this formulation is that it combines in one model most of the known life-stress relationships for one or two types of stresses (such as the Temperature-Non Thermal model), as well as the multivariable proportional hazards model. This yields a single general likelihood function (for a given distribution) whose solution is independent of both the chosen life-stress relationship and the number of stress-types. In addition, this model allows for simultaneous analysis of continuous, categorical and indicator variables. The solution to this model provides the engineers with an opportunity to expand their selection of types of stresses and test conditions when testing products.

1. INTRODUCTION

Accelerated tests are becoming increasingly popular in today's industry due to the need to quickly obtain life data. However, the rate of increase of this popularity has been very slow due to the limited options available at this time for performing adequate life data analyses on data obtained from accelerated tests. Most of the tools available today for accelerated life data analyses are limited to performing analyses on data obtained from tests of one or two-stress variables. In most practical applications however, life is a function of more than one or two variables (stress-types). In addition, there are many applications where the life of a product as a function of stress and of some other than stress engineering variable is sought. The theory for such cases has been developed for some time now, the means of its application however, has been a subject of wishful thinking. The models available for Accelerated Life Testing analyses are generally classified into the Statistics based models and the Empirical-Statistics based models. An example of a Statistics based model is the Proportional Hazards model, and an example of an Empirical-Statistics model is the Arrhenius-Weibull model. A general model is adopted in this paper which incorporates most of the widely used life-stress relationships such as Arrhenius and Inverse Power, as well as the Proportional Hazards model. The model can be used for single or multiple accelerated stresses. Under this formulation, the model can be either solved as a Proportional Hazards Weibull model or as another Empirical-Statistical model by performing a simple transformation on the model's covariates. For example, in a multiple stress type accelerated test the engineer can now choose which types of stresses follow an inverse power relationship, and which follow an exponential relationship when performing the analysis. In addition, indicator and categorical variables can also be included in the analysis, and thus accounting for variables such as material, vendors, etc. Isolating the effects of these factors becomes essential in estimating their influence. In order to do so, these factors must be identified and quantified by numeric variables known as covariates, diagnostic or explanatory variables.

Introduced by D. R. Cox, the Proportional Hazards (PH) model was developed in order to estimate the effects of different covariates influencing the times-to-failure of a system. The need for such a model emerged from the common problem of parts being tested under different conditions. According to the PH model the failure rate of system is effected not only by its operation time, but also by the covariates under which it operates. For example, a unit may have been tested under a combination of different accelerated stresses such as humidity, temperature, voltage, etc. It is clear then, that such factors affect the failure rate of a unit. Utilizing the Maximum Likelihood estimation method, the likelihood function was formulated and solved for complete and suspended data and for Weibull and Lognormal as the underlying life distributions. Confidence bounds on the estimates were quantified using the Variance-Covariance matrix. Two formulations will be presented in this paper. The first formulation is the Proportional Hazards model, and the second is the general Log-Linear model. The Proportional Hazards formulation is in fact a special case of the general Log-Linear model (Refs. 4 and 3).

2. NOMENCLATURE

β	Weibull shape parameter.
η	Weibull scale parameter.
$\Phi()$	is the standard normal cumulative distribution function.
$\overline{T}'$	mean of the logarithm of time-to-failure.
$\sigma_{T'}$	standard deviation of the logarithm of the time-to-failure.

T_F	exact time-to-failure.
T_S	right censored time-to-failure.
m	number of stress related variates (time independent).
X	is a row vector consisting of the covariates.
C	column vector of regression coefficients.
$\lambda_0(t)$	baseline failure rate.
$\lambda(t,\mathbf{x})$	failure rate of the Proportional Hazards model.

3. ASSUMPTIONS

The following to assumptions are made prior to formulating the model:

1. Statistically independent covariates.
2. Common shape parameter. This assumption is valid when similar modes of failure are accelerated.

4. MODEL FORMULATION

Accelerated Life models consist of an underlying failure distribution and a Life-Stress relationship. A percentile of the failure distribution is chosen to be represented by the stress-life relationship. The objective then becomes to obtain the parameters of the failure distribution and the stress-life relationship. In this paper, the Weibull and Lognormal distributions will be considered and the Maximum Likelihood parameter estimation method will be utilized. A general likelihood function is to be formulated for these two distributions. The likelihood function is given by (Ref. 7),

$$L = \prod_{i=1}^{F} f(T_{F,i}) \prod_{j=1}^{S} R(T_{S,j}). \tag{1}$$

Taking the natural logarithm of eq. (1) simplifies the optimization. The Log-Likelihood function is given by (Ref. 7),

$$\ln(L) = \Lambda = \sum_{i=1}^{F} \ln\left[f(T_{F,i})\right] + \sum_{j=1}^{S} \ln\left[R(T_{S,j})\right] \tag{2}$$

The model will be formulated in such a way where eq. (2) will be a function of the stresses by expressing the *pdf* and reliability functions in terms of these stresses.

4.1 *Proportional Hazards (PH)*

The Proportional Hazards model is important in the analysis of life data. The model has been widely used in the biomedical field (Refs. 3 and 2), and recently there has been an increasing interest in its application in reliability engineering. In its original form the model is nonparametric, i.e., no assumptions are made about the nature or shape of the underlying failure distribution. In this paper a parametric form of the model will be considered utilizing a Weibull life distribution.

The Proportional Hazards model assumes that the failure rate of a unit is the product of a baseline failure rate, $\lambda_0(t)$, which is a function of time only, and a positive function (also known as link function) g(**x**, **c**), independent of time, which incorporates the effects of a number of co-variates such as humidity, temperature, pressure, voltage, etc. The failure rate of a unit is then given by,

$$\lambda(t,\mathbf{x}) = \lambda_0(t) \cdot g(\mathbf{x},\mathbf{c}),$$

where,

$$\mathbf{x} = (x_1, x_2, \ldots x_m).$$

c is a column vector consisting of the unknown parameters (also called regression coefficients) of the model,

$$\mathbf{c} = (c_1, c_2, \ldots c_m)^T$$

It can be assumed that the form of g(**x**, **c**) is known and $\lambda_0(t)$ is unspecified. Different forms of g(**x**, **c**) can be used. However, the exponential form is mostly used due to its simplicity and is given by,

$$g(\mathbf{x},\mathbf{c}) = e^{\mathbf{c}^T\mathbf{x}^T} = e^{\sum_{j=1}^{m} c_j x_j}$$

The failure rate can then be written as

$$\lambda(t,\mathbf{x}) = \lambda_0(t) \cdot e^{\sum_{j=1}^{m} c_j x_j} \tag{3}$$

4.1.1 *The PH Weibull Model (PHW)*

The Weibull distribution can be used as the underlying life distribution. In other words it is assumed that the baseline failure rate in eq. (3) is parametric and given by the Weibull distribution. In the case the baseline failure rate is given by,

$$\lambda_0(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1}$$

The PH failure rate then becomes,

$$\lambda(t,\mathbf{x}) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} \cdot e^{\sum_{j=1}^{m} c_j x_j} = \frac{\beta}{\eta^\beta} t^{\beta-1} \cdot e^{\sum_{j=1}^{m} c_j x_j}$$

It is often more convenient to define an additional covariate, $x_0 = 1$, in order to allow the Weibull scale parameter raised to the beta (shape parameter) to be included in the vector of regression coefficients. The PH failure rate can then be written as:

$$\lambda(t,\mathbf{x}) = \beta \cdot t^{\beta-1} \cdot e^{\sum_{j=0}^{m} c_j x_j}$$

The reliability equation can be derived as,

$$R(t,\mathbf{x}) = e^{-\int_0^t \lambda(u,\mathbf{x})du} = e^{-t^\beta \cdot e^{\sum_{j=0}^{m} c_j x_j}} \tag{4}$$

The *pdf* can be obtained by taking the partial derivative of the reliability function given by eq. (4) with respect to time. The reliability function and the *pdf* can then be substituted into eq. (2). This yields the likelihood function for the PHW model as follows:

$$\Lambda = \sum_{i=1}^{F} \ln\left(\beta \cdot T_{F,i}{}^{\beta-1} e^{\sum_{k=0}^{m} c_k x_{i,k}} e^{-T_i^\beta \cdot e^{\sum_{j=0}^{m} c_j x_{i,j}}}\right) - \sum_{k=1}^{S} T_{S,k}{}^{\beta} \cdot e^{\sum_{j=0}^{m} c_j x_{k,j}} \tag{5}$$

Solving for the parameters that maximize eq. (5) will yield the parameters for the PHW model. Note that for $\beta = 1$ eq. (5) becomes the likelihood function for the PH exponential model, which is similar to the original form of the proportional hazards model proposed by Cox (Refs. 2 and 1).

4.2 *Log-Linear Relation*

The formulation of the model begins with the assumption of a log-linear relation for the nominal life, t_p, as follows

$$\ln(t_p) = a_0 + a_1 x_1 + \ldots + a_m x_m. \tag{6}$$

The coefficients of eq. (6) are the parameters of the stress-life relationship. The variables x_1, x_2,…, x_m are the covariates. These covariates can represent indicator variables, categorical variable, stress, or some transformation of stress.

Eq. (6) can be rewritten in a more compact form as,

$$\ln(t_p) = \sum_{k=0}^{m} a_k x_k ,$$

or, in terms of the nominal life,

$$t_p = e^{\sum_{k=0}^{m} a_k x_k} . \tag{7}$$

The nominal life, t_p, can represent any percentile. The percentile is chosen according to the assumed underline distribution. The advantage of representing the percentile with the log-linear relationship is that relationships such as the Arrhenius and Inverse Power Law can be assumed for the covariates by performing a simple transformation. The appropriate transformations for some widely used life-stress relationships are given in Table 1.

Table 1: Covariate transformations for some common Life-Stress Relationships

Relationship	Model	Transformation
Arrhenius	$t_p = Ae^{\frac{B}{V}}$	$x = \frac{1}{V}$
Inverse Power	$t_p = \frac{1}{K \cdot V^n}$	$x = \ln(V)$
Combination 1 (Temperature-Non Thermal)	$t_p = \frac{C \cdot e^{\frac{B}{V}}}{U^n}$	$x_1 = \frac{1}{V}$ $x_2 = \ln(U)$
Combination 2 (Temperature-Humidity)	$t_p = Ae^{\frac{\phi}{V}+\frac{b}{U}}$	$x_1 = \frac{1}{V}$ $x_2 = \frac{1}{U}$

4.2.1 *Weibull Distribution*

For the Weibull distribution the chosen percentile is the 63.2% which corresponds to the scale parameter. Eq. (7) for the Weibull distribution is written as

$$\eta = e^{\sum_{k=0}^{m} a_k x_k} . \tag{8}$$

Note than only when beta equals 1, the scale parameter of the Weibull distribution corresponds to the exponential distribution's mean life, which is the parameter of the exponential distribution.

The *pdf* and reliability function for the Weibull distribution is given by

$$f(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} e^{-\left(\frac{t}{\eta}\right)^{\beta}} \tag{9}$$

$$R(t) = e^{-\left(\frac{t}{\eta}\right)^{\beta}} . \tag{10}$$

The reliability equation for the Weibull distribution in terms of the covariates can be obtained by combining eqs. (8) and (10)

$$R(t) = e^{-\left(\frac{t}{e^{\sum_{k=0}^{m} a_k x_k}}\right)^{\beta}} = e^{-t^{\beta} e^{-\beta \sum_{k=0}^{m} a_k x_k}} . \tag{11}$$

The *pdf* of the Weibull distribution in terms of the covariates can be obtained by combining eqs. (8) and (9)

$$f(t) = \beta \cdot t^{\beta-1} e^{-\beta \sum_{k=0}^{m} a_k x_k} e^{-t^{\beta} e^{-\beta \sum_{k=0}^{m} a_k x_k}} . \tag{12}$$

The log-likelihood function given by eq. (2) can now be expressed in terms of the covariates by substituting eqs. (11) and (12)

$$\Lambda = \sum_{i=1}^{F} \ln\left[\beta \cdot T_{F,i}{}^{\beta-1} e^{-\beta \sum_{k=0}^{m} a_k x_{i,k}} e^{-T_{F,i}{}^{\beta} e^{-\beta \sum_{k=0}^{m} a_k x_{i,k}}}\right] - \sum_{j=1}^{S} (T_{S,j})^{\beta} e^{-\beta \sum_{k=0}^{m} a_k x_{j,k}} . \tag{13}$$

The set of parameters which maximize eq. (13) are obtained by simultaneously solving the following partial derivatives

$$\frac{\partial \Lambda}{\partial \beta} = 0 \; ; \; \frac{\partial \Lambda}{\partial a_k} = 0$$

The likelihood function given by eq. (13) has a general form which depending on the transformation on the covariates, different known models can be considered. For example, by performing the transformation given by eq. (14), eq. (13) becomes the likelihood for the Arrhenius relationship.

$$x_k = \frac{1}{S_k} \tag{14}$$

where S_k is the k^{th} stress.

Similarly, the Inverse Power Law relationship can be obtained by performing the transformation given by eq. (15).

$$x_k = \ln(S_k). \tag{15}$$

Note that the likelihood function given by eq. (13) is very similar to the likelihood function for the Proportional Hazards Weibull model given by eq. (5). In particular, the shape parameter of the Weibull distribution can be included in the regression coefficients of eq. (13) as follows:

$$c_i = -\beta \cdot a_i . \tag{16}$$

In this case the likelihood functions given by eqs. (5) and (13) are identical. Therefore, if no transformation on the covariates is performed, the parameter values that maximize eq. (13) also maximize the likelihood function for the Proportional Hazards Weibull (PHW) model with parameters given by eq. (16). Note that for $\beta = 1$ (exponential underlying distribution), eqs. (5) and (13) are identical, and $c_i = -a_i$.

The algorithm developed is independent of the form of the likelihood function. In other words, engineers can switch between the PHW model and the Log-Linear model and perform the analysis either way. In addition, when the Log-Linear model is used, the desired transformation can be assigned to each of the covariates beforehand, without manually transforming the input data.

4.2.2 *Lognormal Distribution*

When the Lognormal is assumed as the underlying life distribution, it is more convenient to consider the median life as a function of the covariates. Eq. (7) is written for the Lognormal distribution as

$$\breve{T} = e^{\sum_{k=0}^{m} a_k x_k}. \tag{17}$$

The *pdf* and reliability function for the Lognormal distribution are given by eqs. (18) and (19).

$$f(t) = \frac{1}{\sigma_{T'} t} \phi\left(\frac{\ln(t) - \bar{T}'}{\sigma_{T'}}\right) \tag{18}$$

$$R(t) = \Phi\left(\frac{\ln(t) - \bar{T}'}{\sigma_{T'}}\right) \tag{19}$$

The log-mean is related to median by the following equation:

$$\bar{T}' = \ln(\breve{T}) \tag{20}$$

Combining eqs. (17) and (20) yields

$$\bar{T}' = \sum_{k=0}^{m} a_k x_k. \tag{21}$$

Using eq. (21), eqs. (18) and (19) can be expressed in terms of the covariates.

$$f(t) = \frac{1}{\sigma_{T'} t} \phi\left(\frac{\ln(t) - \sum_{k=0}^{m} a_k x_k}{\sigma_{T'}}\right) \tag{22}$$

$$R(t) = \Phi\left(\frac{\ln(t) - \sum_{k=0}^{m} a_k x_k}{\sigma_{T'}}\right) \tag{23}$$

And the log-likelihood function using eqs. (22) and (23) becomes

$$\ln(L) = \Lambda = \sum_{i=1}^{F} \ln\left[\frac{1}{\sigma_{T'} T_{F,i}} \phi\left(\frac{\ln(T_{F,i}) - \sum_{k=0}^{m} a_k x_k}{\sigma_{T'}}\right)\right] + \sum_{j=1}^{S} \ln\left[\Phi\left(\frac{\ln(T_{S,j}) - \sum_{k=0}^{m} a_k x_k}{\sigma_{T'}}\right)\right]. \tag{24}$$

5. THE ALGORITHM

Several algorithms have been used in the past for the maximization of the log-likelihood function such as the Newton search [Refs. 2,4], genetic algorithms, annealing method, etc. The problem was approached with a form of the Newton search method, which is closely related to the Quasi-Newton method [Ref. 6]. This method was chosen because it is versatile, reliable, and provides quick convergence. The method maximizes the log-likelihood function (eq. (13) or eq. (24)) by taking Newton steps in order to bring its partial derivatives to zero. The full Newton step is always performed, since a quadratic convergence can be achieved once near the solution. At each iteration it is checked if the proposed step reduces the log-likelihood function. If not, a backtrack along the Newton direction is performed until an acceptable step is achieved. As in every optimization algorithm, the initial guesses for the parameters are very crucial. For this reason, much of the research focused in obtaining them. The initial guesses are obtained from the supplied data, thus increasing the probability of convergence to a global minimum (if it exists), and decreasing the number of iterations. The algorithm was developed to solve for up to eight different stress-types.

6. EXAMPLES

Consider the data summarized in Table 2 and Table 3. These data illustrate a typical 3 stress type accelerated test. Different scenarios will be considered for their analysis in order to illustrate some typical applications of the model.

Table 2: Stress Profile Summary

Profile	Temperature (K)	Voltage (V)	Operation Type
A	358	12	On/Off
B	358	12	Continuous
C	378	12	On/Off
D	378	12	Continuous
E	378	16	On/Off
F	378	16	Continuous
G	398	12	On/Off
H	398	12	Continuous

Table 3: Failure Data

Time	Stress Profile	Time	Stress Profile	Time	Stress Profile	Time	Stress Profile
498	A	211	D	145	F	116	H
750	A	266	D	192	F	149	H
445	B	298	D	208	F	155	H
586	B	343	D	231	F	173	H
691	B	364	D	254	F	193	H
20 Units Suspended at 750		387	D	293	F	214	H
176	C	14 Units Suspended at 445		10 Units Suspended at 300		7 Units Suspended at 228	
252	C	118	E	87	G		
309	C	163	E	112	G		
398	C	210	E	134	G		
		249	E	163	G		

6.1 *Temperature*

To illustrate the use of the model, the data of Table 3 will be first analyzed as a temperature stress test, assuming a Weibull distribution and an Arrhenius stress-life relationship. For this case, only the temperature effect will be considered as if the data were obtained from a one stress-type accelerated test. From Table 2, the test was performed at three different temperatures.

Eq. (7) then becomes

$$\eta = e^{a_0 + a_1 \frac{1}{S_1}}$$

The resulting relationship after performing these transformations is in the form of the Arrhenius relationship.

$$\eta = e^{a_0} e^{a_1 \frac{1}{S_1}} = A e^{\frac{B}{S_1}}$$

Therefore, the constant, *A*, of the Arrhenius relationship can be obtained by:

$$A = e^{a_0},$$

and the parameter *B* is equal to the Log-Linear coefficient a_1.

The estimated parameters obtained using the developed algorithm are (Refs. 9 & 10):

β = 2.9479; a_0 = -5.9204; a_1 = 6071.43; a_2 = -1.5604.

Once the parameters are estimated, further analysis on the data can be performed. A very common plot for the analysis of accelerated data is the Life vs. Stress plot, as shown in Figure 1. When the Arrhenius relationship is assumed, this type of plot is called an Arrhenius plot. In such a plot life is plotted for a range of stress levels in a Log-Reciprocal scale. Each line in Figure 1 is a constant reliability line.

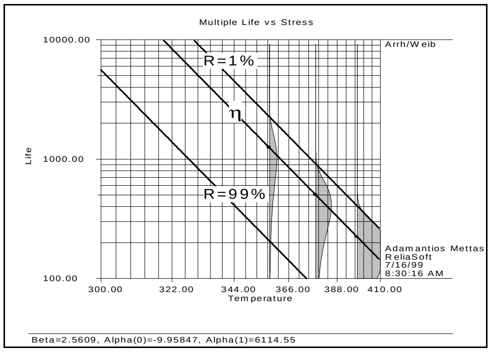

Figure 1: Life vs. Stress plot (Arrhenius Plot)

6.2 *Temperature-Voltage*

In this case, the data will be treated as if the temperature and voltage were the only accelerated stresses in the test. This will illustrate the use of the model when two variables are considered for transformation. The Arrhenius relationship will be used to describe the temperature stress, and the Inverse Power to describe the voltage.

Eq. (7) then becomes

$$\eta = e^{a_0 + a_1 \frac{1}{S_1} + a_2 \ln(S_2)}$$

The resulting relationship after performing these transformations is a form of the Generalized Eyring relationship (Ref. 4).

$$\eta = e^{a_0} e^{a_1 \frac{1}{S_1}} e^{a_2 \ln(S_2)}$$

$$= e^{a_0} e^{a_1 \frac{1}{S_1}} S_2{}^{a_2}$$

The best fit values for the parameters in this case are (Refs. 9 & 10):

β = 2.9479; a_0 = -5.9204; a_1 = 6071.43; a_2 = -1.5604

Several types of information about the model as well as the data can be obtained from a probability plot. For example, the choice of an underlying distribution and the assumption of a common slope (shape parameter) can be examined (Ref. 8). In this example, the linearity of the data supports the use of the Weibull distribution (Figure 2). In addition, the data appear parallel on this plot, therefore reinforcing the assumption of a common beta. Further statistical analysis can and should be performed for these purposes as well. An extensive coverage of assessing the model assumptions is provided by Nelson [4].

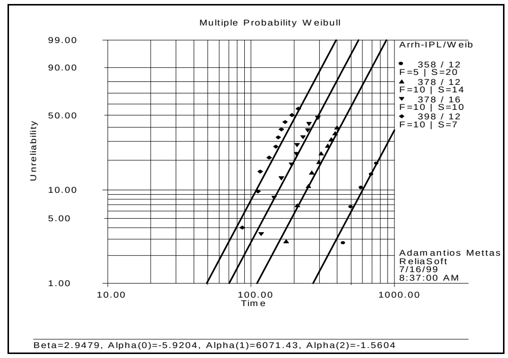

Figure 2: Probability Weibull plot for the two-stress test case

Life vs. Stress plots can be very useful in assessing the effect of each stress to a product's failure. In this case, since the life is a function of two stresses, two different plots should be created. Such plots are created by holding one of the stresses constant at the desired use level, and varying the second. For example, Figure 3 was generated by holding the voltage constant at 10V, and plotting life as a function of Temperature. Three life lines were chosen, namely the characteristic life, η, and the 1% and 99% reliabilies. Similarly, and for a use temperature of 328K, Figure 4 was generated. In addtion the 2-sided 90% confidence bounds for the 99% reliability line were plotted in Figure 4. This provides the flexibility of obtaining the 99% life for any voltage level with the corresponding confidence bounds. The two plots can be compared and an assessment of the effect of each stress on the life of the product can be made, by looking at the typical operation ranges for each stress. Additional statistical methods for this purpose can also be performed (Ref. 3).

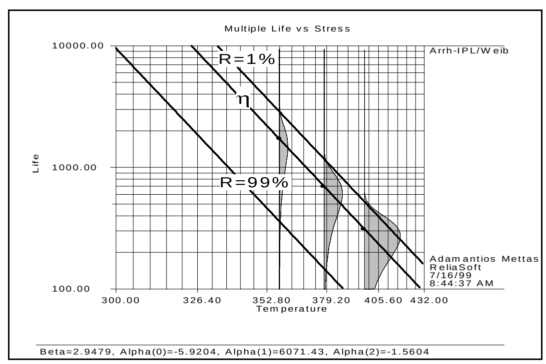

Figure 3: Life vs. Temperature plot for a use voltage of 10V

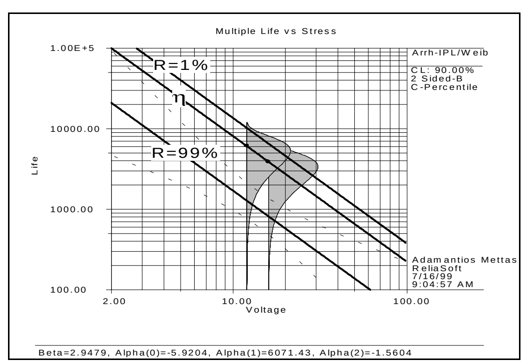

Figure 4: Life vs. Voltage plot for a use temperature of 328K

6.3 *Temperature-Voltage-Operation Type*

In this case all the variables are considered. The same transformations are performed for temperature and voltage, and no transformation is performed on the operation type. The operation type variable is treated as an indicator variable, taking the discrete values of 0 and 1, for On/Off and Continuous operation, respectively. The best fit values for the parameters in this case are (Ref. 9):

$\beta = 3.74830$; $a_0 = -6.02201$; $a_1 = 5776.94047$;
$a_2 = -1.43404$; $a_3 = 0.62424$

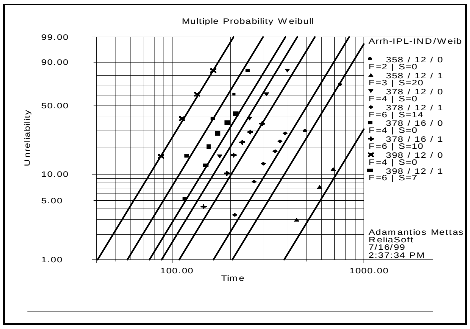

Figure 5: Probability Weibull plot for all the covariates

The effects of the two different operation types on life can be observed on Life vs. Stress plots. In Figure 6 the scale parameter vs. temperature was plotted for the two operation types and for a constant voltage of 10V. The top line represents life under the continuous operation scenario, and the bottom one is the On/Off case. It can be seen that the On/Off cycling has a greater effect on the life of that product in terms of accelerating failure than the continuous operation.

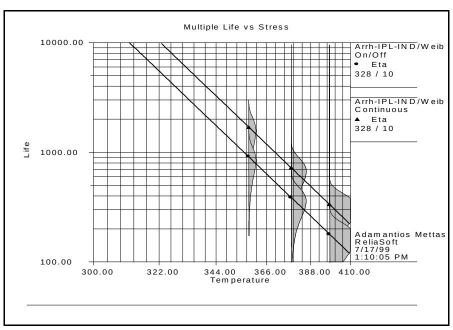

Figure 6: Life vs. Temperature plot for a use voltage of 10V

N. H. Teng et.al. [Ref. 5] provide a case study where a Proportional Hazards Weibull model was fitted to the life data of small dc motors.

7. CONCLUSIONS

A mathematical formulation and its solution were presented in this paper. The combination of the two gives engineers the opportunity of performing data analysis to more than one stress-type accelerated tests. Design of Experiments methods can also be used more frequently and with much more flexibility when designing accelerated tests. The failure distribution model enables assessment of product reliability at any desired stress level, as well as for combination of stresses. The effect of the multiple stresses on the life of the product can then be easily and quickly quantified and fully assessed. In addition, the flexibility of selecting a specific accelerated relation for each stress-type is also provided, by performing a simple transformation on the covariates. Life testing usually requires long time at the lower stress levels. In such cases very few or no failures are observed. Since all data is important, all suspensions should be considered. Using the Maximum likelihood estimation method all data are considered for the analysis, even data from stresses and stress levels where no failures were observed. It should be noted that the acceleration equation is a quantitative relationship between the acceleration stresses and life. This equation is best determined by engineering knowledge and physics of failure and validated by experience. Nelson [4], states "every relationship needs to be supported in application by data and experience, since theory is merely theory." In addition, multiple stress-type accelerated tests and analyses should be performed with caution, since the introduction of many variables has its shortcomings. Larger sample sizes and formal experimental design methods should be used for sound results of both the accelerated test and analysis.

8. ACKNOWLEDGMENTS

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BIOGRAPHY

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Mr. Mettas is a Research Engineer at ReliaSoft Corporation in Tucson, Arizona. He has coordinated ReliaSoft's theoretical research efforts and formulations in the subjects of Accelerated Life Testing and System R&M. Mr. Mettas holds a *BS* Degree in Mechanical Engineering and an *MS* Degree in Reliability Engineering from the University of Arizona. His research interests include life data analyses, system reliability, maintainability, simulation, accelerated testing, and R&M software.